

# **Engineering Economics**

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Schmid, B., Vanderby, S.

Mitch Cassidy, Mila Markevych, Riley Gyorf

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# Signposting

## Engineering Economics

This textbook received funding to employ three students from the University of Saskatchewan to work on this book with us. We are very thankful for the effort they have put into developing sections of the book and in many cases providing some levity in the example problems. Thank you Riley, Mitch, and Mila!

Note: While this text is available, some minor text edits and updates of figures may occur from time to time.

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# Terminology

Throughout this textbook, we will use a number of technical and shorthand terms to explain business, economic, and financial concepts. The terms are explained as they are introduced in the textbook chapters, but a number of them will be listed here so that the reader may become familiar with them before plunging into the technical topics or refer to this section for reminders.

- **Principal:** the amount of money involved in the debt or investment activity
- **Interest Rate (i):** the cost of borrowing money expressed as a percent over a time period
- **Discount rate (i):** used in place of an interest rate when calculating future or present values in an economic analysis
- **Interest period:** the frequency that interest is calculated (eg. daily, weekly, monthly, annually)
- **Duration/Length:** the total period of time that the loan/investment applies to. It is normally expressed in the total number of interest periods (N). Also known as the *amortization period* (for loans) or *interest term*.
- **Receipts & Disbursements:** money received and spent that make up a particular cash flow pattern over a specific length of time.
  - Receipts (or income): +ve
  - Disbursements (or expenses): -ve
- **Future value of money (F):** the result of the cumulative effect of earning interest over a number of interest periods
- **Present value (P):** the amount that a future sum of money is worth today, given a specific discount rate.
- **Total Interest (I):** the total amount of money earned (or paid) over a period of time due to the interest rate.
- **Uniform Series:** a series of equal payments, (A), that occur over a number of consecutive periods. Also called an **equal-payment series** or an **annuity**.
- **Project:** used sometimes to refer to the schedule of cash flows associated with a project or investment.

Other points:

- When we say interest, we typically mean compound interest. Assume compound interest unless stated otherwise or simple interest is stated.

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# Importance of Economics in Engineering

There are few engineering projects undertaken that do not have the cost of the project as a major, if not the most important, factor in the decision to undertake the project. The cost of the project is weighed against the potential for profit or the benefits that the project may provide. While we could envision projects in our personal lives such as remodeling our houses, or deciding if its more economical to repair our car versus buying a replacement, the projects we undertake as engineers are often much larger in scope and cost. The scale of projects in engineering can be in the 100's of millions of dollars for significant infrastructure projects; consider, for example, the electricity grid and modernization for renewable energy sources, the transportation network, and natural resource projects such as mines, mills, and petroleum facilities.

While it is true that a lot of the work done by engineers involves problems solving and designing solutions to problems; typically these solutions are constrained by completing the design in the most economical way. Economic decisions are different but not separate from design decisions. In design we are working with our knowledge of physics, material properties, chemistry, and design codes to formulate our solutions. In economics we are making decisions based on factors such as market demand for products or services, the price we can offer a product at in comparison to competitors, and changes in the inputs such as commodity prices. We may complete the economic analysis many times with different assumptions to determine how to proceed.

Often we concern ourselves with a comparison of the potential profit of a project relative to its cost to build and operate. Other factors are now becoming much more important in the evaluation of projects; social and environmental considerations are becoming as important as the economic considerations and need to be included in our overall analysis of projects.

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# Canadian Perspectives and Tax Laws

This book provides a discussion and examples using the Canadian tax system when dealing with depreciation calculations. Each country has its own set of laws governing taxes and it is left to the reader to understand the tax implications in their own jurisdiction. General methods for considering the depreciation of assets over the life of the asset are valid in many regions.

In Canada, tax information is provided through the Canada Revenue Agency and information about the tax system can be found on their website (<https://www.canada.ca/en/revenue-agency.html>). This is a good resource for the classification of assets for the purposes of calculating depreciation and determining the capital cost allowance for tax deduction calculations.

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# Cash Flows and Cash Flow Diagrams

**Cash flows** are a fundamental tool in engineering economic analysis. Cash flows represent transactions in which money changes hands between two or more parties – lending \$10.00 to a friend or making a payment on a car loan, for example. Representing transactions as cash flows makes it easier to keep track of the important information included in the transactions. Specifically, there are two characteristics of financial transactions that are indicated in cash flows:

- **Value** – the magnitude of the transaction being described. This is dependent on two factors: the *amount* of money or currency changing hands (a dollar value) and the *direction* in which the money is flowing (the orientation of the cash flow). We represent financial gains (also called receipts or income) as positive in value, and financial losses (also called disbursements or expenses) as negative in value.
- **Timing** – the time or period in which the cash flow occurs. Often, periods are set to coincide with interest periods, which will be discussed further in the Time Value of Money chapter. Typically, periods are in increments of months, quarters (1/4 of a year), semi-annual, or annual but other time increments may also be used.

Let's use the example of someone lending \$10.00 to a friend. We'll assume Riley lends his friend Chris \$10.00 on January 1, and Chris pays back the \$10.00 on February 1. There are two transactions in this example: the initial lending on January 1, and the repayment on February 1. Each can be represented by its own cash flow. From Riley's perspective, as the lender, the two cash flows would look like so:

**Table 1: Cash flows from Riley's Perspective**

Cash Flow	Timing	Value
Lend \$10 to Chris	January 1	-\$10.00
Receive \$10 from Chris	February 1	+\$10.00

The table in Table 1 gives a concise summary of the two cash flows involved in the example, from the lender's perspective. But how would things change if we made the same table from Chris's (the borrower's) perspective?



**Table 2: Cash flows from Chris' Perspective**

Cash Flow	Timing	Value
Receive \$10 from Riley	January 1	+\$10.00
Repay \$10 to Riley	February 1	-\$10.00

As we can see by comparing Tables 1 and 2, changing perspectives changes the signage of the cash flows. This is because when one party gives money to the other, the recipient's total assets increase, and the donor's assets decrease.

The previous example shows how cash flows can be used to summarize the important information in financial transactions. When conducting an analysis in a spreadsheet it is common to list cash flows in tabular format. When trying visualize or explain the financial transactions in a particular analysis, cash flows can be represented in a much simpler way – the **cash flow diagram**. The next section covers cash flow diagrams in detail.

**Cash Flow Diagrams** **Cash flow diagrams** are simple graphical representations of financial transactions. The diagrams consist of arrows, such as in the diagram shown below.

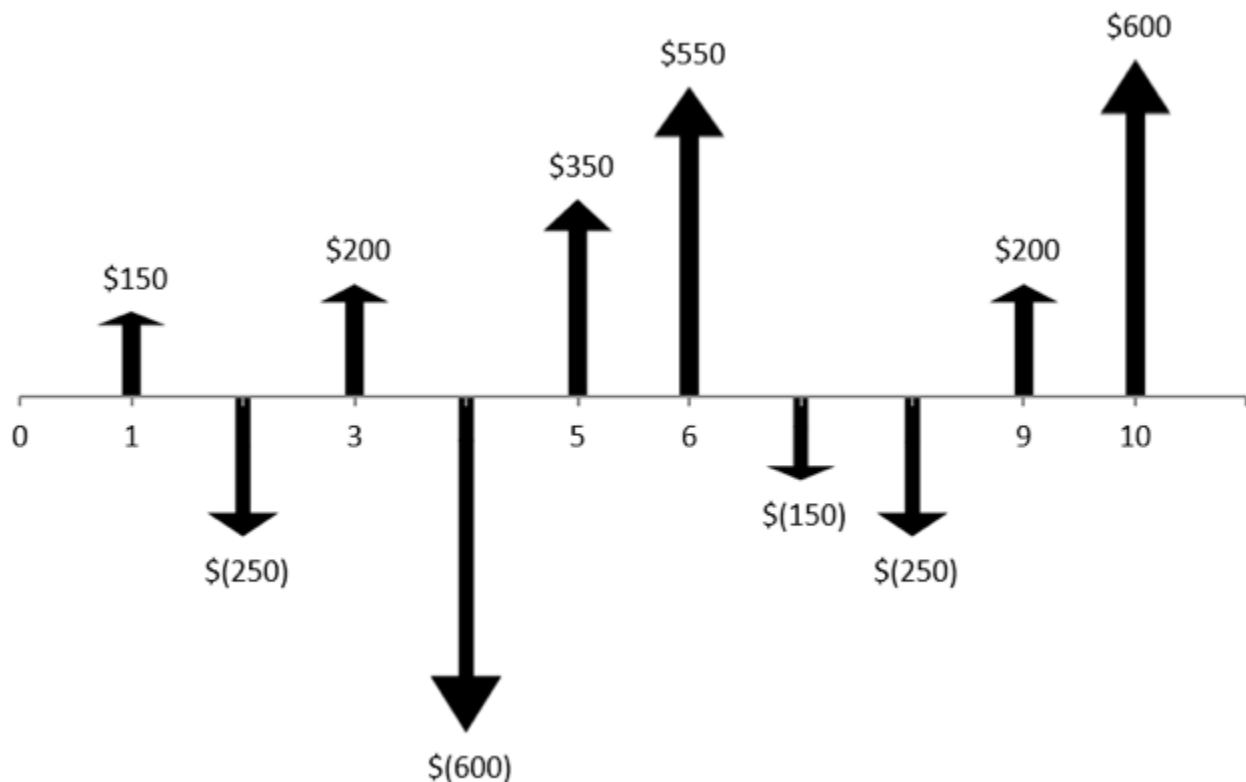


Figure 1 Cash Flow Diagram

Cash flows like the ones shown in the above figure are typical. There are some basic rules for creating cash flow diagrams:

- Time is represented by a horizontal line marked with the number of **periods** in the analysis. The choice of time interval will reflect the project or transactions being considered.
- The horizontal position of each arrow indicates the timing of that cash flow.
- Upward arrows represent **positive** cash flows, also known as inflows, income, or receipts.
- Downward arrows represent **negative** cash flows, also known as outflows, disbursements, or expenses.
- Each arrow represents the **net** cash flow *in that period* (receipts – disbursements). There is only one cash flow arrow for each period representing this net value.

Let's return to the previous example: Riley lending \$10.00 to Chris. We can draw out the cash flows graphically as a cash flow diagram. Just like with the tabular form of the cash flows, we can represent the cash flows in two different diagrams: one from Riley's perspective and one from Chris's perspective. If we set January to be the first period, and February to be the second period, the two diagrams would look like so:

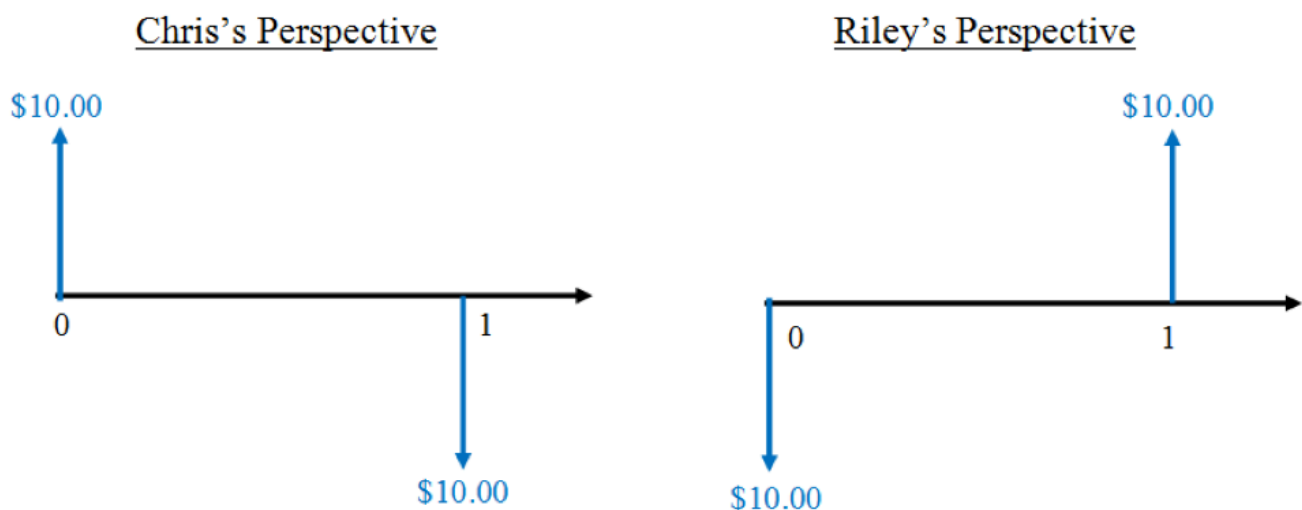


Figure 2: Cash flow diagrams of a lending agreement

These diagrams are really just graphical representations of financial transactions. As one can see, the diagrams for Riley and Chris give the same information as the tables we showed before, but it may be less time-consuming to draw these simple diagrams than it is to write out an entire table! The movement of money is also easier to show graphically. Note that only one of the diagrams is required to represent the problem. You would simply choose to display the project from either Riley's perspective or from Chris's perspective.

Note that this difference in perspective does not affect the equivalence of cash flows (equivalence is covered in detail in the Time Value of Money chapter). The different perspectives affect the timing and orientation of the cash flows, but the net effect remains the same. In this example, both parties ended up with no net loss or gain after Riley was repaid, because no interest was charged on the loan. If Riley had charged Chris \$1.00 in interest, then Riley would have earned a net profit of \$1.00 and Chris would have lost \$1.00 in total. The money is all accounted for, regardless of who the observer is.

## End-of-Period Convention

In practice, cash flows can occur at any time within a period. However, for simplicity, we commonly assume the **end-of-period convention** – the assumption that all cash flows occurring within a period are moved to the end of the period. The following figure graphically demonstrates the end-of-period convention.

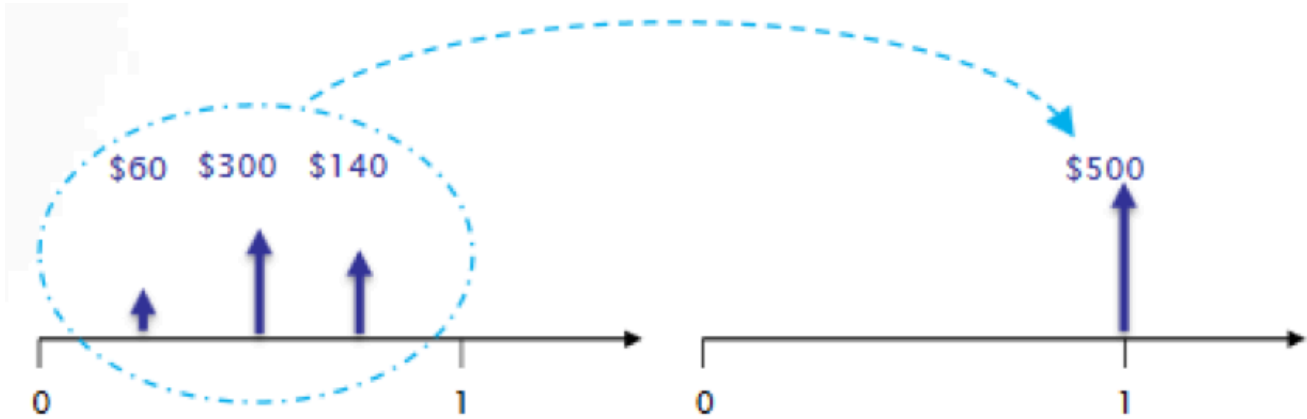


Fig. 3: End of Period Diagram

In the figure above, there are several cash flows at different times within the same period. Using the end-of-period convention, we sum these cash flows together and move them to the end of the interest period as one net cash flow.

While this assumption could introduce some discrepancies between the model and real-world results, it simplifies calculations greatly, as we will see in later chapters. The difference between the model results and real-world results are generally minor and insignificant to the analysis.

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# Chapter 1: Cost Estimation and Budgeting

## Introduction

Suppose you're planning a Thanksgiving dinner for your family, and you want to budget for the cost of the meal. You could make a list of the ingredients needed for each of the dishes (turkey, stuffing, cranberry sauce, etc.) and then check a flyer from a local grocery store for the price of each item. If you wanted to go into extreme detail, you could even calculate the increase in your electric bill from cooking your turkey in the oven (although you would likely decide this is too negligible to consider). The point is, with a little bit of effort, you could estimate a fairly accurate budget for your meal.

Now imagine that you're planning to build a 2000 MW capacity hydroelectric dam. The reservoir created above the dam will hold trillions of litres of water, the dam structure will require an enormous amount of material to construct, and the entire project will take years to complete (the Hoover Dam was completed over 5 years between April 20, 1931 and March 1, 1936). How would you develop a budget for this project?

Engineers regularly deal with projects whose scale, complexity, and uncertainty call for sophisticated approaches to planning and budgeting. This chapter will give a basic introduction to the cost-identification and budgeting methods that can be applied to engineering projects.

## Key Concepts

- Overhead costs
- Cost driver identification
- Cost allocation methods – activity-based and departmental
- Capital budgeting
- Flexible budgeting
- Zero-based budgeting
- Budgets and estimates

**Learning Objectives**

After completing this chapter, students should be able to:

- Identify and calculate manufacturing overhead costs
- Understand the relationship between activities and cost drivers
- Recognize different cost-allocation methods and their advantages and disadvantages
- Apply the methods to assign costs and determine the total cost of a product
- Understand budgeting and its role in planning, control, and decision making
- Make reasonable financial estimates and use them in planning/budgeting decisions

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## 1.1 Costing

A **cost** is a price that must be paid for goods and services that are expected to benefit or be required for a project. When determining or assigning costs to specific elements of a project, we say that we are **costing** these elements. Any project or project element for which a cost is determined is referred to as a **cost object**. Understanding the various types of costs that face projects is critical for effective budgeting and project management. Companies can use cost information in several ways, such as to evaluate their performance and increase the efficiency of their business, to set prices, to bid on new projects, and to inform important management decisions like investing in new equipment, changing suppliers, or discontinuing existing projects.

Due to the complexity of engineering projects, we typically categorize costs so that they are easier to identify and analyze. There are several different ways to select these categories, depending on how the budget is going to be used.

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## 1.2 Basic Budgeting

A **budget** is an estimate of the revenues and costs associated with a project over a specified period of time. Engineers use budgets to *plan* the future actions required for a project and their associated costs, and to *control* the progress of a project by comparing it with the plan over time. This means that budgets are an important part of both short term and long term planning for any substantial financial investment, and indispensable when monitoring and managing engineering projects. Budgeting forces project managers to plan ahead, improves their decision making resources, provides a standard for performance evaluation, and improves communication and coordination on a project.

There are several classifications and styles of budgets, which will be discussed in more detail in Section 6. To begin, it is most important to understand the format and function of any basic budget. Budgets list quantities, costs, and revenues based on estimates of future activity, and combine those estimates to predict the total financial situation of a project, department, or company.

To better understand this, let's look at this basic example of a quarterly budget for a restaurant:

<b>Sales</b>		
	Restaurant Sales	\$ 155,200
	Take-out Sales	\$ 218,880
<b>Cost of Sales</b>		
	Food Costs	\$111,360
<b>Profit From Sales</b>		<b>\$ 222,720</b>
<b>Payroll</b>		
	Chefs	\$ 26,880
	Servers	\$ 80,640
	Delivery	\$ 40,320
<b>Overhead</b>		
	Lease	\$ 8,000
	Utilities	\$ 3,000
	Depreciation	\$ 2,000
<b>Total Additional Expenses</b>		<b>\$ 160,840</b>
<b>Net Profit</b>		<b>\$ 61,880</b>

One of the first things to notice is that budget begins by listing its income, which in this case is generated from food sales. Although food costs could be listed as an expense, since they are so closely related to the restaurant's income they are grouped in the same "sales" section. The "profit from sales" budget line subtracts the cost of sales from the sales revenue and shows the total profit for the business before additional expenses. The second half of the budget lists expenses, broken into payroll for its employees and additional operational costs referred to as "overhead". (Note: "overhead costs" are covered in more detail in Section 3.2.) The "total additional expenses" line sums all of the business expenses listed, with the exception of food costs, as previously mentioned. The "net profit" for the business during this quarter is the difference between the profit from sales and the total additional expenses. In this case, the difference is positive, so the restaurant is *expected* to generate income during this quarter.

Note that budgets combine both known and estimated values, thus they may not be completely accurate. Some values, just as the monthly cost of leasing the restaurant may be known in advance, as it is likely based on a long-term contract. Other costs, such as the delivery and food costs, as well as the expected revenues are likely based on estimates. These should be well-informed estimates however, either well-researched or based on experience, thus are hopefully quite accurate. Yet there is still an element of "guess work" involved, as various factors (many beyond the control of the restaurant owners) can affect the actual values. For example, a new restaurant may open nearby, attracting some of their patrons, thus lowering their revenues, but also lowering food and utility costs as they would correspondingly prepare and cook fewer meals.



We are now familiar with the layout of a basic budget, but to create our own we need to understand how the costs that make a budget are determined and organized.

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## 1.3 Categorizing Costs

The example shown above is quite simple and only provides summary values, hiding many of the details underlying the values shown (e.g. the number of servers, their associated working hours and hourly wages are likely used to estimate the total “servers” payroll cost.) Real-world budgets can get very complex, thus it is useful to organize and categorize activities, materials, costs, etc. in a logical manner to both help ensure that the budget is comprehensive and thoroughly considers all factors involved with the overall endeavour, and to help accurately and transparently communicate the budget to others. There are several different ways costs can be categorized, as discussed in the following sections.

### 1.3.1 *Fixed, Variable, and Mixed Costs*

One way to group costs within a budget is to look at **fixed costs** versus **variable costs**. Variable costs will increase proportionally with the output volume of a project (thus they vary) while fixed costs will remain constant regardless of output volume. Consider the cash flows for a grain bin manufacturer; in order to sell more bins, the company must produce more bins, and therefore spend more on raw materials. However, regardless of how many bins it produces, the company will still spend the same amount each month to lease its production building and storage yard. Thus, we can say that the raw materials represent a variable cost, and the lease represents a fixed cost.

Suppose that the same manufacturer has a salesperson who markets the grain bins to local farmers. The salesperson is paid a base salary for the year, but to incentivize them to sell more bins they also make a small commission from each sale they make. Since the cost to pay this salesperson is partially fixed, and partially variable based on their output, we say that it is a **mixed cost**. Other common mixed costs include water and electricity costs, which often include both delivery costs, which are fixed, and usage costs, which vary based on consumption.

These cost categories are commonly used when projecting costs as they enable comparison between different production levels. For example, the grain bin manufacturer could compare expected profits for normal production versus a 10% increase, and so on.

It should be noted that many fixed costs are only truly fixed for a certain range of activity and dramatic changes may alter these as well. If the grain bin manufacturer decided to double production, their storage yard may not have adequate capacity and they may have to lease additional storage area to accommodate the increased

volume of units in inventory, thus the fixed cost of their lease would also increase. In general, though, if altering production by one unit (either an increase or a decrease) would not affect a cost, we would consider it fixed. If it would affect a cost, we would consider it variable.

### 1.3.2 Direct Costs, Indirect Costs, and Overhead

Another way to categorize costs is distinguish between direct costs and overhead costs. A **direct cost** is any cost which can be completely and directly assigned to a specific cost object. For example, in a manufacturing context, the two major sources of direct costs are materials and labour; the direct cost for a manufactured product would therefore be the cost of its material components plus the labour cost to assemble it. Since these costs can be directly traced to the cost object being produced, they can be assigned accurately without additional effort. However, even in basic manufacturing situations, not all costs can be directly assigned to a cost object.

Suppose you are managing the budget of a small local brewery. You might find that several of the major cash flows are straightforward to identify: the brewery gets its revenue from selling beer, and pays direct costs for its raw ingredients. However, there are many other unrelated expenses which must be considered, including heat and lighting, equipment maintenance costs, marketing, rental costs or property taxes for the facility, among others. All these costs must be taken into account in the selling price of the brewery's products. As they affect the company's profitability, they cannot be ignored. As such, we refer to any cost which is not directly related to a specific product as an **indirect cost**. Even the salaries of the brewery employees may be indirect costs: the wages for people on the production line are direct costs, whereas the wages for administrative staff (among others) are indirect costs.

To make this more complicated, there is a third category of costs known as **overhead**, which is often a subset of indirect costs. You can think of overhead costs are costs which would be incurred regardless of whether any product is produced, and typically are long term costs (i.e. they continue for as long as a product is in production). Other indirect costs may be incurred only once during the life of a product; these are typically indirect, but not overhead, costs.

For example, a direct cost of a pharmaceutical drug would be the chemicals that go into making a pill. An overhead cost of the drug would be the utilities to run the company's production lab. An indirect, but not overhead, cost may be the cost of the clinical trial for that pill.

The lines between overhead and indirect costs are fuzzy. Some sources consider the two terms to be synonymous, and some draw slightly different distinctions than the one given above. Due to the lack of distinction, we will use these terms interchangeably in this text.

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## 1.4 Cost Driver Identification

Although general overhead costs are not associated with a specific product, we still need to incorporate them into product costs in a rational way. To do this, we look for some way to allocate these costs to a product; that is, we try to determine a way to divide the overhead costs to each product we're making in some way that is a fair representation of the proportion of the overhead costs each product "caused".

Suppose you ran a sporting goods manufacturing plant which makes both tennis balls and golf balls. To understand the true cost of making each product, you would want to make sure that you accurately represent its share of the plant's electricity bill. You produce an equal number of each product, but since the golf balls are more energy intensive to produce, they account for 90% of the power consumption at your plant. Logically, we would want to attribute 90% of the electricity cost to the golf ball product, and only 10% to the tennis balls. This is an example of how we can assign overhead costs using a **cost driver**.

A **cost driver** is a factor that allows overhead costs to be reasonably assigned to a cost object. The cost driver must either be the cause of the overhead costs, or at least reasonably associated. If you were estimating the cost of a road trip, the length of the trip might be an appropriate cost driver to use. Although the actual cost would vary depending on the fuel efficiency of your vehicle, the traffic conditions, your driving habits, and the price of gas, you could roughly say that if your vehicle uses 12 litres per 100 km on the highway, and gas costs about \$1 per litre, that you will spend 12 cents per kilometer for your trip. This cost driver allows you to easily estimate the cost for a trip from Saskatoon to Whitehorse (2500km -> \$300), or Vancouver to Halifax (6000km -> \$720), or any other trip with a simple calculation.

For a manufacturing situation like our sporting goods factory, we might use the common cost driver of machine-hours to estimate power consumption. The more hours the golf ball machines operate to produce each ball, the greater the cost assigned to the product. While this may not produce a perfect estimate (suppose the golf ball machines consume slightly more power each hour compared to the tennis ball machines), it is clear that this cost driver is still closely related to the overhead cost in question, and should produce some kind of reasonable estimate.

Let's look at an example which will combine what we have learned about direct and indirect costs:

EXAMPLE

The Not-Flat-Earth Telescope Company has a production plant which manufactures several different models of telescopes to fill orders for stores around the province. Two of its most popular models are the Myopiscope and the Hyperscope. The material and manufacturing costs associated with making one telescope of each model are shown below. The plant is a large facility for which overhead costs include its lease, utilities, machine maintenance, and general administration. To ensure that the revenue from telescope sales covers these costs, management assigns overhead to each model of telescope based on three separate cost drivers. For each machine-hour used, it assigns \$10 to cover utilities (e.g. heat, electricity) and machine maintenance. It also charges an additional 5% of the materials cost to account for their indirect raw material procurement costs (e.g. shipping and receiving department), and 25% of the labour cost to cover the salaries of support staff (e.g. sales, human resources). Not-Flat-Earth also pays an average wage of \$18/hour for its direct labour costs.

	Myopiscope	Hyperscope
Optical Tube	\$4.00	\$6.50
Lenses	\$15.00	\$35.00
Mount	\$3.00	\$4.00
Labour Hours	1	2
Machine Hours	0.5	0.6

This morning, Not-Flat-Earth sold an order of 30 Myopiscopes and 12 Hyperscopes to the Sell-O-Scope Astronomy Shop. How much should they have charged for this order?

Step 1: Calculate the direct cost for a single telescope of each model, based on materials and labour.

$$DC = \text{Optical Tube} + \text{Lenses} + \text{Mount} + (\text{labour hours} \times \text{hourly wage})$$

$$DC_{\text{Myopiscope}} = \$4.00 + \$15.00 + \$3.00 + \left(1 \text{ hour} \times \frac{\$18}{\text{hour}}\right) = \$40.00$$

$$DC_{\text{Hyperscope}} = \$6.50 + \$35.00 + \$4.00 + \left(2 \text{ hours} \times \frac{\$18}{\text{hour}}\right) = \$81.50$$

Step 2: Calculate the additional overhead costs for a single telescope of each model.

$$\text{OH} = (\text{Machine Hours} \times \text{Hourly rate}) + (\text{material Cost} \times \text{\%rate}) + (\text{Labour Cost} \times \text{\%rate})$$

$$\text{OH}_{\text{Myopiscope}} = \left(0.5 \text{ hours} \times \frac{\$10}{\text{hour}}\right) + (\$4 + \$15 + \$3) \times 5\% + \left[\left(1 \text{ hour} \times \frac{\$18}{\text{hour}}\right) \times 25\%\right]$$

$$\text{OH}_{\text{Myopiscope}} = \$10.60$$

$$\text{OH}_{\text{Hyperscope}} = \left(0.6 \text{ hours} \times \frac{\$10}{\text{hour}}\right) + (\$6.5 + \$35 + \$4) \times 5\% + \left[\left(2 \text{ hours} \times \frac{\$18}{\text{hour}}\right) \times 25\%\right]$$

$$OH_{Hyperscope} = \$17.28$$

Step 3: Find the total cost of each telescope by combining its direct and overhead costs, and then multiply to get the total cost for the order.

$$\text{Cost} = \text{DC} + \text{OH}$$

$$Cost_{Myopiscope} = \$40.00 + \$10.60 = \$50.60$$

$$Cost_{Hyperscope} = \$81.50 + \$17.28 = \$98.78$$

$$\text{Order Cost} = (30 \times \$50.60) + (12 \times \$98.78) = \$2703.36$$

**So, we find that Not-Flat-Earth should have charged Sell-O-Scope \$2703.36 for the order.**

---

## 1.5 Cost Allocation Methods

Once you've applied a costing procedure to determine the costs of a project, the next step in making a good budget is to allocate those costs into appropriate categories. This can be done in many ways, but two common approaches are **function-based costing** and **activity-based costing**. This section will introduce you to using both of these approaches.

### 1.5.1 Function-Based Costing

**Function-based costing** works by using production drivers (e.g. machine hours, labour hours) to assign costs to various departments (e.g. sales, marketing, manufacturing). This is the method that businesses have traditionally used to track their costs, since it provides a straightforward breakdown of where money is spent within the company, and it provides good information about a business's function for external investors. We can think of function-based costing as a three step process:

#### 1. Identify Unit-Level Drivers

Function-based costing is highly dependent on the concept of *units* and *unit costs*, where the unit is the basic quantity of the good or service being produced. For a shoe factory the unit would be a pair of shoes, and for a tour company a unit might be a single 1-hour helicopter tour. The cost drivers used in this costing method are exclusively "unit-level drivers"; that is, they are variable costs, highly correlated to the number of units produced. All overhead costs which are variable costs (e.g. utilities) are assigned using driver tracing. Any overhead costs which are fixed costs, or are not correlated to unit production (e.g. building lease, equipment depreciation), must be allocated on a different basis.

In order to determine accurate unit costs, the first step must be identifying the unit-level drivers of those costs. As examples, these drivers could include machine-hours, labour hours, or simply the number of units produced.

#### 2. Estimate Driver Capacity

Once you have identified the unit-level drivers for each department, you must estimate its capacity, or number of units that you expect to be produced over a certain period. You might find it useful to consider various types of estimates, as illustrated here with the example of a gas station:

**Expected Capacity:** On any given day, the gas station expects to sell about 12,000 litres of gasoline, based on an average from the last full year of sales. The expected capacity is a short term estimate of capacity.

**Normal Capacity:** If gas prices are unusually low this year, the gas station might expect gas consumption to dip again if prices rise back to their typical levels. Therefore, they might expect to sell about 11,000 litres of gasoline per day over the next 5 years. The normal capacity is a longer term estimate of capacity.

**Theoretical Capacity:** Suppose the city was hit with a hurricane warning, and every car in the city rushed to the station to fuel up so that they could leave town. From opening to close, every pump in the gas station would be occupied, and they could sell every last drop of their 50,000 litre fuel reservoir in a single day. The theoretical capacity is an absolute maximum for capacity.

**Practical Capacity:** If the station operated with high efficiency, so that even during peak hours of the day there were always open pumps and no customers were turned away to the competing gas station across the road, they could sell 14,000 litres of gas per day. The practical capacity is an estimate of the maximum capacity under efficient operation.

### 3. Estimate Overhead Cost

The final step in function-based costing is to estimate overhead costs. First, the total overhead costs for the operation must be determined. Then, these overhead costs can be used to determine a department-wide overhead rate based on a unit level driver. Finally, these overhead costs can be assigned to individual products based on the amount of the unit level driver used per product .

Let's try an example to see how function-based costing can be applied:

#### EXAMPLE

*Not-A-Real-Mountain Manufacturing produces two models of downhill skis: the Faceplant and the ShowOff. The production of the skis is a two-step process, beginning in the Production Department and finishing in the Assembly Department. The table below lists the direct labour hours (DLH), machine hours (M/C Hours), total direct costs, and overhead costs incurred during manufacturing 10,000 sets of skis. After identifying the most relevant cost drivers, the company decides that overhead should be applied to the production process on the basis of machine hours, and to the assembly process on the basis of direct labour hours. Using the data given:*

1. a) determine an appropriate overhead rate per set of skis, for both models, and
2. b) find the total amount that should be charged for an order of 700 Faceplants and 250 ShowOffs, including direct costs and overhead.



	Production			Assembly		
	Faceplant	Show-off	Total	Faceplant	Show-off	Total
<b>No. of units</b>	8000	2000	10000	8000	2000	10000
<b>Direct Labour hrs</b>	450	190	640	2850	1250	4100
<b>M/C hours</b>	1300	420	1720	350	150	500
<b>Direct Cost</b>	\$192,000	\$94,000	\$286,000	\$104,000	\$68,000	\$172,000
<b>Overhead</b>		\$65,000			\$24,000	

Part a)

Step 1: Begin by finding plant-wide overhead rates for production and assembly, by dividing our total overhead costs by the total machine hours and total direct labour hours, respectively.

$$OH_{Production} = \frac{\text{Total Production Overhead}}{\text{Total Production M/C Hours}} = \frac{\$65,000}{1720 \text{ hours}} = \$37.79 \text{ per hour}$$

$$OH_{Assembly} = \frac{\text{Total Assembly Overhead}}{\text{Total Assembly DLH}} = \frac{\$24,000}{4100 \text{ hours}} = \$5.85 \text{ per hour}$$

Step 2: Assign these numbers based on the number of machine hours or labour hours expended on each unit to find production and assembly overhead rates for an individual set of each model of skis.

$$OH_{FaceplantProd} = Rate_{Production} \frac{\text{Faceplant M/C Hours}}{\text{No. of Faceplant Units}} = \$37.79/\text{hour} \frac{1300 \text{ hours}}{8000 \text{ units}} = \$6.14/\text{unit}$$

$$OH_{ShowoffProd} = Rate_{Production} \frac{\text{Showoff M/C Hours}}{\text{No. of Showoff Units}} = \$37.79/\text{hour} \frac{420 \text{ hours}}{2000 \text{ units}} = \$7.94/\text{unit}$$

$$OH_{FaceplantAssembly} = Rate_{Assembly} \frac{\text{Faceplant DLH}}{\text{No. of Faceplant Units}} = \$5.85/\text{hour} \frac{2850 \text{ hours}}{8000 \text{ units}} = \$2.08/\text{unit}$$

$$OH_{ShowoffAssembly} = Rate_{Assembly} \frac{\text{Showoff DLH}}{\text{No. of Showoff Units}} = \$5.85/\text{hour} \frac{1250 \text{ hours}}{2000 \text{ units}} = \$3.66/\text{unit}$$

Step 3: Find total overhead rates by combining the production and assembly rates for each model.

$$OH_{Faceplant} = OH_{FaceplantProd} + OH_{FaceplantAssembly} = \frac{\$6.14 \text{ unit} + \$2.08}{\text{unit}} = \$8.22/\text{unit}$$

$$OH_{Showoff} = OH_{ShowoffProd} + OH_{ShowoffAssembly} = \frac{\$7.94 \text{ unit} + \$3.66}{\text{unit}} = \$11.60/\text{unit}$$

**The overhead rate for the Faceplant model is \$8.22/unit, and for the ShowOff model is \$11.60/unit.**

Part b)

Step 1: Find the direct cost per unit for each model, combining production and assembly costs.

$$DC_{Faceplant} = \frac{DC_{FaceplantProd} + DC_{FaceplantAssembly}}{\text{No. of Faceplant Units}} = \frac{\$192,000 + \$104,000}{8000 \text{ Units}} = \$37/unit$$

$$DC_{Showoff} = \frac{DC_{ShowoffProd} + DC_{ShowoffAssembly}}{\text{No. of Showoff Units}} = \frac{\$94,000 + \$68,000}{2000 \text{ Units}} = \$81/unit$$

Step 2: Find the price for the order in part b) using the direct cost and overhead cost per unit.

$$Price = Units_{Faceplant} (DC_{Faceplant} + OH_{faceplant}) + Units_{Showoff} (DC_{Showoff} + OH_{Showoff})$$

$$Price = 700 \text{ units } (\$37/unit + \$8.22/unit) + 200 \text{ units } (\$81/unit + \$11.60/unit)$$

Total Cost = \$54,804

**The total amount that should be charged for this order of skis is \$54,804 plus a markup for profit.**

### 1.5.2 Activity-Based Costing

Activity-based costing uses resource drivers (e.g. time, space) and activity drivers (e.g. units produced) to assign costs to various activities. Here we define an **activity** as some specific action or set of related actions that perform work for a project, such as shipping or receiving. Since this method breaks down costs by the specific activities of the company, rather than by department, which it makes it a good ideal approach for internal decision making by showing because it shows the resources used and income generated by the individual activities of the business. This approach is newer than function-based costing, and can also be more time consuming to carry out, because it may require costs from several departments to be estimated and recombined. We can think of activity-based costing as a four step process :

#### 1. Identify Activities

There may be many possible ways to differentiate and categorize the activities completed by a certain department. To ensure that you make useful distinctions, it is usually helpful to interview managers or representatives of the department in question, who will be most familiar with the work done in their area. With this information, you can develop an “activity dictionary” which tracks the specifics of each activity, including its attributes, metrics (drivers), and who it is used by. Depending on the complexity of a business and the level of detail in the analysis, you may choose to record hundreds of activities.

#### 2. Assign Costs to Activities

Once you have arrived at a list of activities, the next step is determine a cost for each one based on the resources which it consumes. As with function-based costing, these costs can either be directly apparent, or determined with the use of cost drivers if costs are split between several activities. This will create an “activity rate” which represents the cost of performing the activity.

#### 3. Assign Activity Costs to Other Activities

This is an intermediate step, wherein any activities whose costs are dependent on other activities are assigned those costs in turn. For example, if one activity is “advertising” and another is “sales and marketing”, the cost of advertising might be added as one part of the sales and marketing costs.

#### 4. Assign Costs to Products

Finally, the cost of a product can be found by summing the activity costs that contribute to its production. These activity costs are found by multiplying activity rates by the *practical capacity* (the amount an activity is used) of their activity, and determining the amount of each activity used by a given product.

Let’s took a look at a question similar to our previous example, this time using activity based costing.

#### EXAMPLE

*Totally-A-Real-Mountain Manufacturing produces two models of downhill skis: the SafeSlope and the Widowmaker, which are manufactured in the same department. The table below lists the direct labour hours (DLH), machine hours (M/C Hours), total direct costs, and overhead costs (which are broken down into four activities: machining, maintenance, assembly, and testing) for a production run of 10,000 sets of skis. After identifying the most relevant activity drivers, the company decides that machining and maintenance are a function of machine hours, assembly is a function of direct labour hours, and testing is a function of the number of units produced. Using the data given:*

1. a) *determine an appropriate overhead rate per set of skis, for both models, and*
2. b) *find the total amount that should be charged for an order of 600 SafeSlopes and 150 Widowmakers, including direct costs and overhead.*

	SafeSlope	Widowmaker	Total
No. of units	5500	4500	10000
Direct labour hours	300	250	550
M/C Hours	1100	800	1900
Direct Cost	\$165,000	\$139,500	\$304,500
Machining	\$12,000	\$11,500	\$23,500
Maintenance	\$4,000	\$3,500	\$7,500
Assembly	\$10,000	\$1,500	\$11,500
Testing	\$20,000	\$500	\$20,500

Part a)

Step 1: Find the proportion of costs for each of the four overhead-generating activities to the costs of their activity drivers.

$$\frac{\text{Total Machining Cost}}{\text{Total Machine Hours}} = \frac{\$23,500}{1900 \text{ M/C hours}} = \frac{\$12.37}{\text{M/C hour}}$$

$$\frac{\text{Total Maintenance Cost}}{\text{Total Machine Hours}} = \frac{\$7,500}{1900 \text{ M/C hours}} = \frac{\$3.95}{\text{M/C hour}}$$

$$\frac{\text{Total Assembly Cost}}{\text{Total Direct Labour Hours}} = \frac{\$11,500}{550 \text{ DLH}} = \frac{\$20.91}{\text{DLH}}$$

$$\frac{\text{Total Testing Cost}}{\text{Total No. of Units}} = \frac{\$20,500}{10000 \text{ units}} = \frac{\$2.05}{\text{unit}}$$

Step 2: Using the data from the 10,000 unit run, find direct labour hours, machine hours, and direct cost per unit for each model of skis.

$$\frac{\text{SafeSlope Direct Labour Hours}}{\text{No. of SafeSlope Units}} = \frac{300 \text{ DLH}}{5500 \text{ units}} = 0.055 \text{ DLH/unit}$$

$$\frac{\text{SafeSlope Machine Hours}}{\text{No. of SafeSlope Units}} = \frac{1100 \text{ M/C hours}}{5500 \text{ units}} = 0.2 \text{ M/C hours/unit}$$

$$\frac{\text{SafeSlope Direct Costs}}{\text{No. of SafeSlope Units}} = \frac{\$165,000}{5500 \text{ units}} = \$30/\text{unit}$$

$$\frac{\text{Widowmaker Direct Labour Hours}}{\text{No. of Widowmaker Units}} = \frac{250 \text{ DLH}}{4500 \text{ units}} = 0.056 \text{ DLH/unit}$$

$$\frac{\text{Widowmaker Machine Hours}}{\text{No. of Widowmaker Units}} = \frac{800 \text{ M/C hours}}{4500 \text{ units}} = 0.178 \text{ M/C hours/unit}$$

$$\frac{\text{Widowmaker Direct Costs}}{\text{No. of Widowmaker Units}} = \frac{\$139,500}{4500 \text{ units}} = \$31/\text{unit}$$

Step 3: Combine the rates calculated above to find unit overhead costs for the two models.

$$OH_{\text{SafeSlope}} = \frac{\$30}{\text{unit}} + \frac{0.056 \text{ DLH}}{\text{unit}} \left( \frac{\$20.91}{\text{DLH}} \right) + \frac{0.178 \text{ M/C Hours}}{\text{unit}} \left( \frac{\$12.37}{\text{M/C Hour}} + \frac{\$3.95}{\text{M/C Hour}} \right)$$

$$OH_{\text{Widowmaker}} = \$9.87/\text{unit}$$

**The overhead rates for the SafeSlope and the Widowmaker should be \$10.92/unit and \$9.87/unit, respectively.**

Part b)

Step 1: Find the price for the order in part b) by combining direct and overhead costs to get a total unit cost for each model, and then multiplying by the number of units in the order.

Unit Cost = Direct Costs + Overhead Cost

$$\text{Unit Cost}_{\text{SafeSlope}} = \frac{\$30}{\text{unit}} + \frac{\$10.92}{\text{unit}} = \$40.92/\text{unit}$$

$$\text{Unit Cost}_{\text{Widowmaker}} = \frac{\$31}{\text{unit}} + \frac{\$9.87}{\text{unit}} = \$40.87/\text{unit}$$

$$\text{Price} = \left( 600 \text{ units} \times \frac{\$40.92}{\text{unit}} \right) + \left( 150 \text{ units} \times \frac{\$40.87}{\text{unit}} \right) = \$30,682.50$$

**The total price charged for the order should be \$30,682.50 plus a markup for profit.**

Gut Check: Is this a good answer? We followed all of the steps in the right order, so we should expect this amount to cover the total expenses of producing these skis. But look at the assumptions that Totally-A-Real-Mountain made about their activities: do the costs for machining, maintenance, assembly, and testing really correlate with the activity drivers they were assigned? You should notice that the assembly and testing costs for the Widowmaker are far lower than for the SafeSlope, even though their labour costs, direct costs, and units produced were extremely similar. If the costs of these activities were truly related, this should not be the case.

Since we are using one general rate for assembly and testing costs, we will be *overestimating* the overhead cost for Widowmakers, which use these activities far more, and *underestimating* the overhead cost for SafeSlopes, which use these activities far more. The order in part b) requests *four times* more SafeSlopes than Widowmakers, so our underestimation of these overhead costs may mean that Totally-A-Real-Mountain is not charging enough, and actually losing money on this order.

The bottom line is that choosing cost drivers and activity drivers to allocate overhead should be done carefully, because associating costs with unrelated drivers can lead to inaccurate estimates and loss of capital.

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## 1.6 Budget Types

Now that we are more familiar with how costs are determined and categorized, we can return to the overall idea of budgeting in greater detail. In most businesses, the most important, over-arching budget for the entire organization is known as the **master budget**, and represents a comprehensive financial plan [for what time period?] for the entire organization. The master budget is typically composed to two distinct parts: the operating budget and the financial budget. These in turn can be composed of other, smaller budgets representing specific parts of the businesses costs and revenues, as shown in Figure HHHH. This text will not provide a detailed overview of every budget type listed below, but will explain the general functions of operating and financial budgets.

Master Budget	
Operating Budget	Financial Budget
Sales Budget	Cash Budget
Production Budget	Budgeted Balance Sheet
Direct Materials & Purchases Budget	Capital Expenditures Budget
Direct Labour Budget	
Overhead Budget	
Selling and Administrative Expenses Budget	
Ending Finished Goods Inventory Budget	
Cost of Goods Sold Budget	

The **operating budget** shows the incoming-generating activities of an organization . This includes all costs and revenues associated with sales, production, inventory, overhead, etc. All of the budgets we have looked at so far in this chapter have been operating budgets. Businesses will usually prepare the operating budget for their business prior to the financial budget, since elements of the financial budget will be affected by the operating budget.

The **financial budget** shows an overall financial position by detailing the inflow and outflow of cash from a company. The elements of this budget are not directly tied to the income-generation of the company, and include any expected borrowing or investments, and managing the company's assets.

While these examples represent the most common format for budgets used within businesses, there are also other approaches to budgeting which are useful to understand.

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## 1.7 Budgeting Approaches

### 1.7.1 Static and Flexible Budgeting

The budgets which we have looked at so far in this chapter might be called **static budgets**. This means that they predict a certain level of activity based on informed estimates, and then budget based on that level of activity. However, as engineers we know as engineers that the assumptions underlying our estimates carry a degree of uncertainty; the estimates we make even just one a single year in advance may not accurately reflect the amount that your company produces or expends on certain activities. If the actual levels of productivity observed do not match with our estimates, it can be useful to have more flexibility.

**Flexible budgets** still use estimates to prepare for a certain level of activity, but then can be used to continually review the variance between *actual* and *expected* levels of activity. The variance is simply the difference between the budgeted costs and actual costs for any line item on a budget, which can be either favourable or unfavourable depending on how it influences the bottom line of the budget. For example, raw material expenses being lower than budgeted would be unfavourable, but sales volume being lower than budgeted would be unfavourable. When flexible budgeting is applied to an activity based budget, it is known as an activity flexible budget.

The table below shows a flexible budget for the costs on the first month of a construction project. What can we learn from this?



	Budgeted	Actual	Variance
<b>Materials</b>			
Timber Piles	\$4,500	\$8,755	-\$4,255
Concrete	\$10,000	\$9,830	\$170
Rebar	\$700	\$678	\$22
Steel Columns	\$7,500	\$7,534	-\$34
Brick	\$3,500	\$3,606	-\$106
Gravel Fill	\$2,500	\$2,134	\$366
<b>Total Materials</b>	<b>\$28,700</b>	<b>\$32,537</b>	<b>-\$3,837</b>
<b>Payroll</b>			
Managers	\$14,000	\$13,567	\$433
Labourers	\$40,000	\$41,233	-\$1,233
Machine Operators	\$10,000	\$9,902	\$98
<b>Total Payroll:</b>	<b>\$64,000</b>	<b>\$64,702</b>	<b>-\$702</b>
<b>Overhead</b>			
Utilities	\$1,000	\$1,238	-\$283
Fuel	\$2,500	\$2,637	-\$137
Equipment Depreciation	\$1,000	\$1,000	\$0
General Administration	\$3,000	\$2,516	\$484
<b>Total Overhead:</b>	<b>\$7,500</b>	<b>\$7,436</b>	<b>\$64</b>
<b>Total Cost</b>	<b>\$100,200</b>	<b>\$104,675</b>	<b>-\$4,475</b>

For this construction budget, no income is listed, so the unfavourable variances are all negative and marked in red, with the favourable variances marked in black. This flexible budget shows that the project is slightly overbudget, with some estimates being slightly under and some slightly over. For instance, the company might wish to investigate why it underestimated the payroll for its labourers, and whether they were needed to work additional hours or overtime that was not accounted for. However, the clear issue with this budget, which accounts for almost all of the negative variance, is that the actual cost for timber piles was almost double that of the budgeted price. If the company is planning on placing more piles for this project, it should either look for a cheaper supplier or set aside more money for them in subsequent months. Overall, it should be noted that although this budget has a negative variance overall it is still very accurate, since it is only about 4% over budget.

### 1.7.2 Incremental and Zero-Base Budgeting

Imagine that it is your first day on the job as the project manager at an enormous factory for a large and well-

established business. Although you have no previous experience with the business, you are told by the president of the company that your first job is to create a master budget planning the factory's operations for the next year. Where do you begin?!

The most common response would likely be to find the budget from the previous year, and use that as the basis for your new budget. Some small changes and updates would likely be necessary, but for a large, well-established business the master budget is likely to remain very similar from year to year, and it would likely not make sense to start from scratch and compute each cost and revenue source individually. This method of beginning with an existing budget, and making small adjustments for variables like price, sales, and product mix is known as **incremental budgeting**.

The alternative to this method – using no prior budget as a basis, and tallying each cost or activity individually – is known as **zero-base budgeting**. On one hand, we can see how this might be time-consuming and inefficient for the example of the factory master budget given above. On the other, this approach might be very well suited to reviewing discretionary or support activities within that same company. The zero-base budgeting approach requires you to prioritize each activity and justify its associated expenses, which makes it great for reviewing resource allocation and spending priorities. Where incremental budgeting might preserve inefficiencies in a business by transferring them year-to-year into the new budget, zero-base budgeting allows you to catch and eliminate these inefficiencies by investing more time and effort into the budgeting process.

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# Chapter 2: Financial Statements and Taxation

## Financial Statements and Taxation

### Introduction

Nearly every day 23 million Canadians wake up, put on their shoes, and head to work at some point in their week.[1] People work so they can afford the things they want and need. Most of us need to pay for the housing, heating, food, clothing and water we need to survive. Many of us want things, such as TVs, cell phones, and vacations. Businesses are the form in which we achieve this system. Businesses such as hospitals, farms, utility companies, Coca-Cola, Sony, and GM are able to generate billions of dollars by producing goods and services to meet our needs and wants.

Does generating money simply mean that a company is successful? Many companies have generated billions in revenues and still suffered bankruptcy (e.g. Nortel). How do we know a company is actually doing well? If you are comparing two companies each with annual revenues of \$10 million, how do you know which is more profitable, or which you would like to invest in? How is the information on a company's performance recorded and how can it be used to determine a company's profitability? If you run your own business, how would you describe your company's financial performance over the last year?

Imagine you operate a business that specializes in making tires for transport trucks. You sell each set of tires for \$500 and in your first month of business you sell 100 sets; you generated \$50,000! How much did you spend making those tires though? To produce each set, you need to spend \$200 on aluminum alloy and \$50 on rubber. This means you made \$250 from each set, right? You also have to pay rent on the factory in which you produce the tires, salaries for the employees who make the tires as well as the administrative staff, and utilities for plant operations. How does this affect your earnings? Do you have enough money each month to purchase supplies or do you have to borrow to do so? Do you know if you are earning a profit? How would you describe your company's financial performance over the last year to a potential investor? Will you have to pay taxes?

In this chapter, we will find answers to these and other similar questions using financial statements of the

company. We will discover what these statements are, what their purpose is, what information they contain and how it can be used to perform analysis of company's activities. We will also discuss how financial statements can be used in forecasting of a company's performance. We will finish this chapter by examining taxation in Canada.

## Key Concepts

- Financial statements (income statement, balance sheet, cash flow statement)
- Financial ratios (profitability, debt management, liquidity, asset management, market value)
- Pro forma financial statements
- Taxation
- Depreciation

## Learning Objectives

After completing this chapter, students should be able to:

- Understand financial statements and use them to find important financial information
- Use financial ratios to evaluate a company's financial performance
- Understand pro forma financial statements
- Calculate income taxes based on Canadian taxation laws
- Calculate depreciation expenses using the three primary methods and apply CCA rules

[1] <http://www.statcan.gc.ca/tables-tableaux/sum-som/l01/cst01/demo10a-eng.htm>

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## 2.1 Financial Statements

Suppose you decide to start a company to manufacture iron rods for building construction. How do you know if your company is doing well? What are the key business objectives that you have to keep in mind? If you are looking for investors to expand your business, what information do you have to present to them? What data do investors and management use to gauge the company's performance? The answers to these questions lie in the financial statements of the company.

**Financial statements** are official records of the financial standing and activities of a business. These statements contain the data necessary to evaluate financial strength of the company. The two primary objectives of every business are profitability and solvency. **Profitability** is the ability to generate income. **Solvency** is the ability to pay debts as they become due. Unless a business can generate satisfactory income to pay its debts, the business cannot survive. (Hermanson R.)

There are three basic financial statements. Together they present the profitability and strength of a company:

- An **income statement**, sometimes referred to as a *statement of profit and loss*, reflects a company's profitability.
- A **balance sheet**, sometimes referred to as a *statement of financial position*, reflects a company's solvency.
- A **statement of cash flows** shows the cash inflows and outflows for a company over a period of time (e.g. a month, a quarter or a year).

The details reported in the financial statement are related to the goal of the statement. That is, the statement contains only the information needed for the goal of that specific statement, e.g. the income statement only includes data that indicates the profitability of the company for a certain period of time. Although the specific headings and elements included on each statement depend on the company's type of business, they are often very similar and follow accounting standards. Standardized formats were established to ensure company reported accurately honestly and transparently. This also enables comparison of the financials between companies. For example, without standardized formats it could be difficult to compare the financial data of a company in the service industry to the one in manufacturing. The structures of the companies, as well as how they generate income are very different. The standardized nature of financial statements, however, allows anyone to see the key information about the companies' performance in a similar way, despite the differences. This allows the

management team and potential investors to navigate the statements and obtain relevant financial information about the company without having to delve into the intricacies of a company's business.

Financial statements contain a lot of specific terminology. We will be explaining some of the most common terms used on the statements as we go.

An essential part of financial statements are the **notes** that accompany them. These notes are generally located at the end of a set of financial statements. They provide further explanation about various items shown in the statements, or other relevant non-quantitative information. For example, a note may indicate the expected lives of long-lived equipment, or loan repayment terms (Annand, 2016).

Note that in this section the goal is to introduce financial statements and the information they contain. The next section will cover how the information from statements can be used to analyze company's performance.

Financial statements typically report the last two or more periods (e.g. months, quarters, years etc.) of activity to enable easy comparison. We briefly introduce the three basic financial statements in the following subsections. To get a better understanding of each of the statements, we use the 2015 consolidated financial statements of Canadian Tire Corporation, Limited[1].

### 2.1.1 The Income Statement

The **income statement**, also called a *statement of profit and loss*, reports the profitability of a business organization for a given period of time. The period must be indicated on the statement, e.g. start and end date.

In accounting, profitability is measured by comparing the *revenues* earned with the *expenses* incurred:

- **Revenue** is the money earned or resulting from the sale of products or from providing services to customers. This may also be called *sales*.

Companies can also earn money from sources that are not related to the normal business operations. For example, a manufacturer can earn interest on loans, or rental income if a business has extra space they rent etc.

- **Expenses** are the costs incurred by the business. These include the cost of goods and services, as well as other costs incurred in day-to-day business activities.

All revenues and expenses are recorded on the income statement to determine profitability of the company. If the revenues are larger than the expenses, then the company is considered to be profitable. The income that is left after all the expenses are accounted for is termed **income before tax**, as shown in Figure 1.1. We will discuss income before taxes in more detail later in this section.

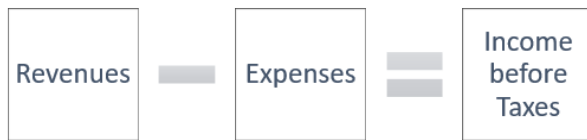


Figure 2.1 *Income before Taxes*

For now, let's take a look at a simplified income statement to see how revenues and expenses are recorded and how income before taxes is calculated. This will give you a basic understanding of how income statements are constructed, and what information is documented.

*Example 2.1 Louri Delivery Inc.*

<b>Louri Delivery Inc.</b>	
<b>Income Statement</b>	
<b>For the Month Ended August 31, 2016</b>	
<b>Revenues</b>	
Service Revenue	\$14,000
<b>Expenses</b>	
Salaries Expense	\$4,800
Rent Expense	\$960
Gas and Oil Expense	\$1,080
<b>Income before Taxes</b>	<b>\$7,160</b>

Example 2.1 shows the income statement of Louri Delivery Inc. The heading shows the name of the entity, the type of financial statement, and the time period the statement covers. The statement lists all revenues and expenses for the period, with the last row stating the income before taxes.

The income statement for the month ended August 31, 2016, shows that the revenues generated by serving customers (or service revenue) for August totaled \$14,000. Expenses for the month include salaries, rent, and gas and oil. To determine the company's income before taxes, the expenses are subtracted from the revenues. Thus, we get:

$$\text{Revenues} = \$14,000$$

$$\text{Total Expenses} = \$4,800 + \$960 + \$1,080 = \$6,840$$

$$\text{Income before taxes} = \$14,000 - \$6,840 = \$7,160$$

So, as a result of business activities in August, 2016, the company's income before taxes was \$7,160. Based on this Louri Delivery Inc. is considered profitable in this period.

*Income before taxes* is one of the so-called **profit indicators** that is noted on income statements. There are other profit indicators that are also commonly recorded (*gross profit*, *operating profit* and *net income*). They are used to determine profitability of the company from different operating perspectives and are commonly associated with the following categories of expenses: *production costs*, *operating expenses* and *non-operating expenses*, as shown in Figure 1.1 below. Combined, these categories account for all of the company's expenses.

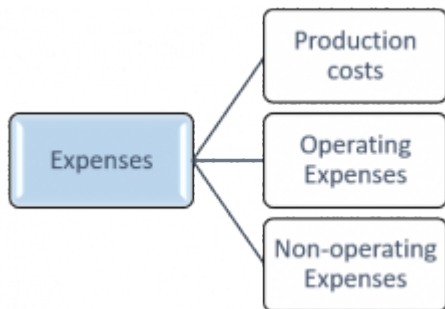


Figure 2.2 Business Expense Categories

**Production costs** are associated with the production of the goods and services sold by the company. In other words, these costs include all the expenses incurred to create the final product or service that goes to the customer. The production costs in a retail business (e.g. car dealership) are called the **cost of producing revenue**. These costs include the amount they pay for the items they sell. The production costs of a manufacturer (e.g. automotive factory) are termed the **cost of goods sold (COGS)**. These costs include the material and labour used to produce the goods. For example, the COGS for a tire manufacturer are the materials (rubber, fabric, wire and other components) and labour costs (wages to workers) used to produce a tire. The cost of transporting the tires to stores and the cost of labour used to sell tires would be excluded as they are not directly attributed to the production of tires.

**Operating expenses** constitute expenditures not directly attributed to production. Just producing goods and services is usually not enough to have a successful business. One must also sell the goods, advertise them, communicate with customers, pay office rent and utilities etc. These costs are not associated directly with production, but are nonetheless important to keep the business *operating*. For example, the operating expenses for a company might include accounting expenses, insurance premiums, office supplies, utilities, advertising etc.

**Non-operating expenses** are incurred outside of a company's core operations. In other words, these are the expenses that are not directly attributed to production or operation of a business. Examples of non-operating expenses are interest paid on a company's loans and money lost due to currency exchange rate fluctuations.

Let's examine some of these expenses using an example of revenues and expenditures of the company Ian's Mats Inc. for July, 2017.

### Example 2.2

Ian has a small business that manufactures rubber mats, Ian's Mats Inc. The company sells these mats for \$68 each. Ian's Mats Inc. produced 100 rubber mats in July, 2017, earning the total sales revenues of \$6800. To



produce one mat, the company needs to spend \$45 on raw materials and labour costs. Ian's Mats Inc. also advertises their mats, spending \$300 this month. When Ian was starting this business, he got a loan to purchase the necessary equipment, and now pays \$35 in interest every month. What would the *income statement* look like for the month ended July 31, 2017?

In this example, the company incurs production expenses, operating expenses and non-operating expenses. Production expenses in this example are the cost of goods sold (COGS). Here,  $\text{COGS} = \$45 \text{ per mat} \times 100 \text{ mats} = \$4500$ . The advertising expenses in the example are categorized as operating expenses. The interest expense is categorized as a non-operating expense. Obtaining and paying interest on the loan is not a primary revenue-generating activity of the company, nor is it an expense incurred to keep the business running (paying interest doesn't directly affect company's performance like advertising does). So, the information that is recorded in the income statement is:

Revenues:

Sales revenues = \$6800

Expenses:

COGS = \$4500

Operating expense = \$300

Non-operating expense = \$35

Total expenses =  $\$4500 + \$300 + \$35 = \$4835$

So, Income before taxes =  $\$6800 - \$4835 = \$1965$ .

Thus, the income statement in this scenario is:

<b>Ian's Mats Inc.</b>	
<b>Income Statement</b>	
<b>For the Month Ended July 31, 2017</b>	
<b>Revenues</b>	
Sales Revenue	\$6,800
<b>Expenses</b>	
COGS	\$4,500
Operating expense	\$300
Non-operating expense	\$4,800
<i>Total Expenses</i>	\$4,835
<b>Income before taxes</b>	\$1,965

As we see, once revenues are recorded on the income statement, the expenses are detailed starting with production costs (*cost of goods sold* or *cost of producing revenue*) and ending with non-operating expenses. This order is commonly used on the income statements to allow for easier calculation of the *profit indicators* mentioned above. We will go into them in more detail, but as a quick overview:

- *Gross Profit* = Revenues – Production Cost
- *Operating Income* = Revenues – Production Cost – Operating Cost
- *Income before Taxes* = Revenues – Production Cost – Operating Cost – Non-operating Cost
- *Net Income* = Revenues – Production Cost – Operating cost – Non-operating cost – Income taxes

The **gross profit** (or **gross margin**) is the first profit indicator listed on an income statement. It reflects the profits remaining after the production costs are deducted from the revenue.

$$\text{Gross profit} = \text{Revenues} - \text{Production Cost} \quad (2.1)$$

The gross profit indicates the profitability of a company's production. For example, if the company's gross profit is negative, that means production expenses exceed revenues from selling company's products. That is, it costs them more to make each item than the selling price, so they are actually losing on every item they produce. This tells the management team to either increase their selling price, or decrease their production costs. This may require substantial changes, e.g. upgrading equipment to increase production efficiency. Other times, the solution may be a small one, like an olive....

An olive?? YES – In the 1980s, Robert Crandall, then head of an airline, cleverly calculated that if you removed just one olive from every salad served to passengers, nobody would notice ... and the airline would save \$100,000 a year! That's one powerful olive! United Airlines, as another example, recently got rid of those "refresher" towels

on most short journeys, cut back on in-flight videos and removed grapefruit juice from its bar menus — it's less popular than orange juice — as part of an overall \$200 million cost cutting program.

Let's go back to example 2.2. What is the gross profit for Ian's Mats Inc. for the month shown? Using equation 2.1 we get:

$$\text{Gross profit} = \text{Revenues} - \text{Production expenses} = \$6800 - \$4500 = \$2300.$$

**Operating profit** is the second profit indicator on an income statement. It reflects the profit remaining after the production cost and operating expenses are deducted from the revenues.

$$\text{Operating profit} = \text{Revenues} - \text{Production Cost} - \text{Operating Expenses}$$

or

$$\text{Operating profit} = \text{Gross Profit} - \text{Operating Expenses} \quad (2.2)$$

Operating profit indicates if the company is making enough money to cover its production and operating expenses. If a company's gross profit is greater than zero, but their operating profit is negative, this means the company isn't able to cover both its production and operating expenses. Somebody isn't going to get paid! So, the company has to increase price, or reduce the costs of production and/or day-to-day business activities. For example, one company in Boston started buying pens and paper in bulk, and asked employees to drink filtered, not bottled, water. Total savings? About \$100,000 per year. At a plant in Texas, it's the doormats in the entryways — now they just get vacuumed instead of sent out for cleaning — and that saves the company \$70,000 a year. Another example — Canon. They saved over \$50,000 using recycled furniture when it expanded an office.

Going back to example 2.2, what is the operating profit for Ian's Mats Inc. for the month? Here, advertising expenses are the operating expenses. Using formula 2.2, we get:

$$\text{Operating profit} = \text{Gross profit} - \text{Operating expenses} = \$2300 - \$300 = \$2000.$$

**Income before taxes**, which we mentioned above, is the third profit indicator on an income statement. It reflects the profit remaining after the production cost, operating expenses and non-operating expenses are deducted from the revenues. Income before taxes is also commonly referred to as *earnings before income tax*.

$$\text{Income before taxes} = \text{Revenues} - \text{Production Cost} - \text{Operating Expenses} - \text{Non-operating expenses}$$

or

$$\text{Income before taxes} = \text{Operating profit} - \text{Non-operating expenses} \quad (2.3)$$

Income before taxes is essentially the company's profits and the amount that is used to calculate the amount

of tax they need to pay. (We will cover income tax in more detail in Section 2.4, but we will briefly note here that tax rates differ between business types and are set by governments.) Income before taxes signals company's profitability when all expenses are accounted for, including the non-operating ones, such as interest they pay on loans. Generally, income before taxes is not significantly different from the operating profit. However, if there is a substantial difference between the two, especially if the operating profit is positive but the income before taxes is negative, their non-operating expenses are high. For example, if the non-operating expenses are high due to a lot of interest paid on their loans, the management of the company might decide to renegotiate some of the loan arrangements or look for alternative financing options.

As shown above in example 2.2, the income before taxes is \$1965. Using formula 2.3 we can calculate income before taxes directly from operating profit:

$$\text{Income before taxes} = \text{Operating profit} - \text{Non-operating expenses} = \$2000 - \$35 = \$1965$$

**Net Income**, is the fourth and main profit indicator on the income statement. It is the income left over after they have paid all of their expenses and taxes. So, it shows whether the company is profitable or not after taxes are paid. If their revenues are higher than their expenses and taxes, they have what we call *net profit*. Otherwise, the business has a *net loss*, and it's losing money. Thus,

$$\text{Net income} = \text{Income before taxes} - \text{Income Taxes} \quad (2.4)$$

Net income is often referred to as a company's "bottom line". It is one of the most followed and talked about numbers when discussing company's performance. It is the money companies can use to finance their operations or expansions, invest in research and development etc., or they can distribute it to the owners.

In example 2.2, suppose Ian's Mats Inc. has to pay \$500 in tax for this month. All the other amounts remain the same as in the original example. What is the net income?

Now that we know the income tax, we can subtract the tax amount from the Income before tax to obtain the net income:

$$\text{Net income} = \text{Income before tax} - \text{Income tax} = \$2000 - \$500 = \$1500.$$

**Now let's look at a real-world example: Canadian Tire's 2015 fiscal year.**

**The table below came from Canadian Tire's website.**

Canadian Tire Corporation, Ltd.		
Income Statement		
For the period ending January 2nd 2016		
	January 2nd, 2016	January 3rd, 2015
For the year ended (C\$ in millions, except for per share amounts)		
<b>Revenue</b>	\$ 12,279.6	\$ 12,462.9
Cost of producing revenue	8144.3	8416.9
<b>Gross margin</b>	4135.3	4046.0
Other (income)	-54.9	-11.0
Selling, general, and administrative expenses	3096.1	3052.9
Net finance costs	92.8	108.9
Change in fair market value of redeemable financial instrument	-	17.0
<b>Income before taxes</b>	1001.3	878.2
<b>Income tax</b>	265.4	238.9
<b>Net income</b>	\$ 735.9	\$ 639.3
Net income attributable to:		
Shareholders of Canadian Tire Corporation	\$ 659.4	\$ 604.0
Non-controlling interests	76.5	35.3
	\$ 735.9	\$ 639.3
Basic Earnings Per Share (EPS)	\$ 8.66	\$ 7.65
Diluted Earnings Per Share (EPS)	\$ 8.61	\$ 7.59
Weighted average number of Common and Class A Non-Voting shares outstanding:		
Basic	76 151 321	78 960 025
Diluted	76 581 602	79 612 957

As you've noticed, this income statement contains terms that may not be clear and are not standard (e.g. "change in fair market value of redeemable financial instrument"). This often arises on financial statements and is one case in which reading the explanatory notes that accompany the statements is helpful. We will skip the entries below the "net income" for now.

### Examining the Income Statement

Let's analyze this statement based on what we've learned. Note that all values are in millions as indicated on the statement.

The income statement shows data for two fiscal years. We will focus on the 2015 fiscal year (column that says "January 2nd, 2015"), which shows:

Total revenue for the year was \$12,279.6.

Total expenses = \$8144.3 + (- \$54.9) + \$3096.1 + \$92.8 = \$11278.3

Income tax = \$265.4.

So, we can calculate net income:

- $Net\ income = \$12279.6 - \$11278.3 - \$265.4 = \$735.9.$

To calculate the other profit indicators on the statement (gross profit, operating profit and Income before taxes), we need to subtract respective expense categories from the revenues.

Using formulas 2.1, we get the gross profit for 2015:

- $Gross\ Profit = Revenues - Cost\ of\ Producing\ Revenue = \$12279.6 - \$8144.3 = \$4135.3$

To calculate Canadian Tire's operating profit using formula 2.2, we need to identify the operating expenses. From their explanatory notes, we know that "selling, general, and administrative expenses" refers to a group of expenses, which includes the cost of marketing products and services, administrative costs, and other general expenses associated with operating a business. So, we categorize "selling, general, and administrative expenses" as operating expenses, which gives us:

- $Operating\ Profit = Gross\ Profit - Operating\ Expenses = \$4135.3 - \$3096.1 = \$1039.2$

To get Income before taxes we use formula 2.3. Here we see that Canadian Tire has listed "other income." This is money they made from sources other than their store sales etc. The amount listed as "other income" is not included in "revenues" amount used to calculate the gross, or operating profit. We only add it when calculating the income before taxes. Since it does not result from the company's core operations, including it in the gross (or operating) profit would exaggerate the profitability of the production or operations of the company.

Here, we see that "other" line shows income in brackets, because the company made money. If the company had lost money, it may have shown "other (expenses)". Note that since it was income in the expense category, the sign is negative in the income statement. This may be confusing, but this is just part of the reporting conventions.

On the next line, we see "net finance costs", which are the costs associated with getting money to finance their operations. Since these costs are not incurred from core operations of Canadian Tire, we categorize them as non-operating expenses. Similarly, the "change in fair market value of redeemable financial instrument" expense is a non-operating expense. So now we can calculate the income before tax as follows:

- $Income\ before\ Tax = Operating\ profit - Non-operating\ expense = Operating\ profit - [- Other\ (income)] - Net\ finance\ costs - Change\ in\ fair\ market\ value\ of\ redeemable\ financial\ instrument =$   
 $\$1039.2 + \$54.9 - \$92.8 = \$1001.3$

So, what does all of this tell us? All profit indicators are positive. This means that Canadian Tire is profitable and operates efficiently on all levels. The bottom line is that they had over a billion dollars in pre-tax income and \$735.9 million in net profit.

### 2.1.2 The Balance Sheet

A **balance sheet** demonstrates a financial position of a company at a specific point in time, e.g. "Balance sheet as of December 31, 2017." This is done by listing a company's assets, liabilities, and stockholders' (owners) equity:

- **Assets** are things of value owned by a business. Assets are usually expected to provide future benefit. They have value because a business can use or exchange them to produce the goods and services, or sell them for profit. Examples include cash, machines, buildings, accounts receivable (amounts due from customers for goods and services previously sold) etc.
- **Liabilities** are the debts owed by a business. Businesses typically incur liabilities by borrowing money (getting a loan) and using credit when purchasing goods and services from suppliers. Most loans have repayment terms and deadlines. Examples of liabilities are loans payable (portion of the loan that is yet to be repaid by the company as of the date of a company's balance sheet), accounts payable (amounts owed to suppliers for previous purchases) etc.
- **Equity** is the value of the business or corporation. It includes retained earnings of the company (accumulated net income retained for company's operation) and the amount of money contributed by the owners (stockholders). The latter is usually recorded as the "capital stock" or "share capital" on a balance sheet, and the money is accumulated by selling *stock* (or *shares*), which are units of ownership in a company in the form of certificates. That is, when purchasing company's shares, an individual (shareholder or stockholder) obtains a right of ownership of a part of the company, and, thus, has the right to claim part of the earnings of the company. The earnings distributed to shareholders are called *dividends*.

These three items are part of the fundamental accounting equation:

$$\text{Equity} = \text{Assets} - \text{Liabilities} \quad (2.5)$$

This equation illustrates the principle of the company's ownership, where equity is equal to what the company owns minus what the company owes.

For example, assume that a company purchased a new vehicle for \$15,000 by using \$10,000 from company's earnings and borrowing \$5,000 from a bank. The price of the asset is \$15,000. \$5,000 is a liability because it is the amount of money you must repay to the bank. Using the accounting equation, we see that:

If the company repays \$2,000 over the next year, and the car stays the same value, what would change in the accounting equation? The asset is still worth \$15,000 but the liability has decreased by \$2,000. What is the company's equity in the car now?

A balance sheet records assets on one side and liabilities and stockholder's equity on the other. Rearranging the accounting equation, we see that:

$$\text{Assets} = \text{Liabilities} + \text{Equity} \quad (2.6)$$

So, the total assets on a balance sheet must equal the total liabilities and equity. This is also why this statement is called the *balance* sheet, because the totals of the two sides of the statement must be equal.

Let's take a look at a simplified balance sheet for Velimir Transportation, Inc. as of December 31, 2017.

*Example 2.3*

<b>Velimir Transportation, Inc.</b>			
<b>Balance Sheet</b>			
<b>As of December 31, 2017</b>			
<b>Assets</b>	<b>Liabilities and Stockholder's Equity</b>		
Cash	\$18,600	<b>Liabilities:</b>	
Accounts receivables	\$650	Accounts payable	\$1020
Trucks	\$40,000	Loans payable	\$18,500
Office equipment	\$3,800	<i>Total liabilities</i>	\$19,520
		<b>Stockholder's equity:</b>	
		Capital Stock	\$35,000
		Retained Earnings	\$8,530
		<i>Total stockholder's equity</i>	\$43,530
<b>Total assets</b>	<b>\$63,050</b>	<b>Total liabilities and stockholder's equity</b>	<b>\$63,050</b>

Notice how the heading of the balance sheet differs from the headings on the income statement (Balance sheet as of December 31, 2017 vs Income Statement for the Period Ending December 31, 2017). A balance sheet is like a photograph; it captures the financial position of a company at a particular point in time. The income statement (and also cash flow statement, which we will cover in section 2.1.3) represents the data over a period of time.

As evident from the statement, the assets of Velimir Transportation, Inc., amount to \$63,050. The company's assets consist of cash, accounts receivable, trucks, and office equipment. Liabilities are the debts owed by a business. Velimir Transportation's liabilities consist of accounts payable and loans payable, totaling \$19,520. The company's stockholders' equity consists of (1) \$35,000 paid for shares of capital stock and (2) retained earnings of \$8,530. Capital stock shows the amount of the owners' investment in the corporation. The total assets (right hand side of the statement) is equal to the total liabilities and stockholder's equity (left hand side of the balance sheet).

**Real-world example: Canadian Tire's 2015 fiscal year.**

**Let's take a look at Canadian Tire's Balance sheet as of January 2, 2016.**



Canadian Tire Corporation, Ltd.		
Balance Sheet		
As of January 2 2016		
as of (C\$ in millions)	January 2nd, 2016	January 3rd, 2015
<b>ASSETS</b>		
Cash and cash equivalents	\$ 900.6	\$ 662.1
Short-term investments	96.1	289.1
Trade and other receivables	915.0	880.2
Loans receivable	4875.5	4905.5
Merchandise inventories	1764.5	1623.8
Income taxes recoverable	42.2	31.9
Prepaid expenses and deposits	96.1	104.5
Assets classified as held for sale	2.3	13.1
<b>Total current assets</b>	<b>8692.3</b>	<b>8510.2</b>
Long-term receivables and other assets	731.2	684.2
Long-term investments	153.4	176.0
Goodwill and intangible assets	1246.8	1251.7
Investment property	137.8	148.6
Property and equipment	3978.2	3743.1
Deferred income taxes	48.1	39.4
<b>Total assets</b>	<b>\$ 14,987.8</b>	<b>\$ 14,553.2</b>
<b>LIABILITIES</b>		
Bank indebtedness	\$ -	\$ 14.3
Deposits	880.7	950.7
Trade and other payables	1957.1	1961.2
Provisions	216.1	206.0
Short-term borrowings	88.6	199.8
Loans payable	655.5	604.4
Income taxes payable	61.5	54.9
Current portion of long-term debt	24.3	587.5
<b>Total current liabilities</b>	<b>3883.8</b>	<b>4578.8</b>
Long-term provisions	45.7	44.1
Long-term debt	2971.4	2131.6
Long-term deposits	1372.2	1286.2
Deferred income taxes	111.1	93.9
Other long-term liabilities	813.9	787.8
<b>Total liabilities</b>	<b>9198.1</b>	<b>8922.4</b>
<b>EQUITY</b>		
Share capital	671.2	695.5
Contributed surplus	2.9	2.9
Accumulated other comprehensive income	148.1	82.0
Retained earnings	4172.0	4075.1
<b>Equity attributable to shareholders of Canadian Tire Corporation</b>	<b>4994.2</b>	<b>4855.5</b>
Non-controlling interests	795.5	775.3
<b>Total equity</b>	<b>5789.7</b>	<b>5630.8</b>

Similarly to Canadian Tire's Income statement, this balance sheet contains a lot of terms that are specific to accounting. To get a better understanding of separate entries on the statement, please refer to the notes that accompany the statements. These can be found on Canadian Tire's webpage. Here, we will briefly go over the structure of the balance sheet without going into much detail as to what individual entries represent.

We will focus on the end of 2015 fiscal year (column that says "January 2nd, 2016"). The balance sheet begins by listing the company's assets. Canadian Tire's assets totaled \$14,987.8 million. Assets are listed as **current** or **long-term** (sometimes referred to as non-current), as shown in Figure 2.2 below.



Figure 2.3 Assets

*Current assets* are expected to be used within a year's time or be readily converted to cash within a year. In other words, they can be liquidated within one year. Liquidity is a measure of how easily an asset can be converted to cash (we will discuss more about liquidity in section 2.2.3). Cash, short-term accounts receivable and investments are examples of current assets and are included on Canadian Tire's balance sheet. Current assets amount to \$8,692.3 million. *Long-term assets* are expected to be useful for longer than a year and are not readily convertible to cash, such as property and equipment, long-term accounts receivable and investments. This distinction between current and long-term assets provides the management of the company the information on how much cash can assets be converted into if the need for immediate cash arises. Here, the company owns \$8,692.3 million worth of current assets that can be converted into cash during one year.

Liabilities are listed next. Canadian Tire's total liabilities are \$9,198.1 million as of January 2, 2016. Like assets, they are listed as either **current** or **long-term**, as shown in Figure 2.3.

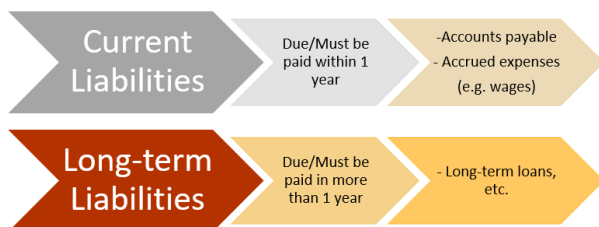


Figure 2.4 Liabilities

*Current liabilities* need to be repaid within one year. For example, a bank loan may need to be repaid in 6 months and accounts payable may need to be paid within one month. Canadian Tire's current liabilities are \$3,883.8 million as of January 2, 2016. *Long-term liabilities* need to be repaid over a longer period of time. Examples

include long-term loans on property and equipment. Interest payments, or other periodic payments, on a long-term liability are considered a portion of current liabilities. The distinction between current and long-term liabilities gives the management of the company the information how much money is needed for the company to repay the liabilities that are due within one year. If the company is able to meet its financial obligations as they become due, the company is considered *solvent*. In this example, the company needs \$3,883.8 million within one year to repay all the current liabilities and remain solvent. As we know, total current assets amounted to \$8,692.3 million, which is enough to repay all the current liabilities of the company, so Canadian Tire is, indeed, solvent. Similarly, when comparing total assets to total liabilities, we see that total assets, \$14,987.8 million, are larger than total liabilities, \$9,198.1 million, which implies that the company is able to meet its financial obligations and is solvent.

Equity is the final section of the balance sheet and includes capital contributed by shareholders and the retained earnings of the company. Equity can be *controlling*, or attributable to the shareholders of the corporation, or *non-controlling*. Non-controlling interest have a financial stake in the company but are not involved in decision making duties. Canadian Tires' equity is equal to \$5,789.7 million.

So, as evident from the balance sheet, Canadian Tire has a good financial position and is a solvent company.

### 2.1.3 The Statement of Cash Flows

Management is interested in the cash inflows to the company and the cash outflows from the company because these determine the company's cash it has available to pay its bills when due. Broadly defined, cash includes both cash and cash equivalents, such as short-term investments.

The **statement of cash flows** shows the cash inflows and cash outflows from *operating*, *investing*, and *financing* activities.

- **Operating activities** generally include the cash effects of transactions and the main operational functions that are used to determine net income.

Cash inflows from operating activities affect items that appear on the income statement and include cash receipts from sales of goods or services, interest received from making loans, and other cash receipts that do not arise from transactions defined as investing or financing activities, such as amounts received to settle lawsuits, proceeds of certain insurance settlements, and cash refunds from suppliers.

Cash outflows include payments to acquire *inventory* (materials, work-in-progress products and finished goods that are part of a company's assets to be sold for profit), to other suppliers and employees for other goods or services, to lenders and other creditors for interest and all other cash payments that do not arise from transactions defined as investing or financing activities, such as taxes and payments to settle lawsuits, cash contributions to charities, and cash refunds to customers.

- **Investing activities** generally include business transactions involving the acquisition or disposal of long-term assets such as land, buildings, and equipment.

Cash inflows from investing activities include cash received from: the sale of property, plant, and equipment, and the collection of long-term loans made to others.

Cash outflows for investing activities include cash paid to purchase property, plant, and equipment, and to make long-term loans to others.

- **Financing activities** generally include the cash effects of transactions and other events involving creditors and owners (stockholders).

Cash inflows from financing activities include cash received from issuing stock (a type of financial instrument that signifies ownership in a company and gives the right to the stockholder to claim part of the company's earnings), mortgages, and from other short- or long-term borrowing.

Cash outflows for financing activities include payments of cash dividends or other distributions to owners (including cash paid to purchase stock) and repayments of amounts borrowed. Payment of interest is not included because interest expense appears on the income statement and is, therefore, included in operating activities.

The cash flow statement reflects a history of cash flows of a company for a given period. It documents the inflows (revenues) and outflows (expenses) of a company's activities. The bottom line of the statement states a company's cash position at the end of the period. This is achieved by adding all the inflows and subtracting all the outflows on the statement.

Management refers to this statement to analyze the inflows and outflows from different company's activities. This statement is also important to consider when studying major policy decisions in the company. When policies are aimed at specific activities (e.g. financing activities), their effect is seen in the form of cash flows from these activities. For example, if the company has optimized its production by using new equipment, the statement may show a significant cash flow from operating activities. If it is large enough, it may be enough to finance all projected capital needs internally rather than having to incur long-term debts. Alternatively, if the company has been experiencing cash shortages, management can use the statement to determine why such shortages are occurring.

Normally, a firm prepares a statement of cash flows for the same time period as the income statement. Let's take a look at a simplified statement of cash flows to see how cash flows from company's activities are recorded. This will give you a basic understanding of how statements of cash flows are constructed.

#### *Example 2.4*

<b>Pere Courier, Inc.</b>	
<b>Statement of Cash Flows</b>	
<b>For the Month Ended September 31, 2016</b>	
<b>Cash Flows from Operating Activities:</b>	
Net Income	\$3,600
<i>Net cash provided by operating activities</i>	<i>\$3,600</i>
<b>Cash Flows from Investing Activities:</b>	
Purchase of Trucks	-\$30,000
Purchase of Office Equipment	-\$1,500
<i>Net Cash Used by Investing Activities</i>	<i>-\$31,500</i>
<b>Cash Flow from Financing Activities:</b>	
Proceeds from Sales of Capital Stock	\$40,000
<i>Net cash provided by financing activities</i>	<i>\$40,000</i>
<b>Net Increase in Cash</b>	<b>\$12,100</b>

Example 2.4 shows a statement of cash flows of Pere Courier, Inc. Similarly to an income statement, the heading of the statement of cash flows shows the name of the entity, the type of financial statement, and the time period the statement covers. The statement lists all cash inflows and outflows for the period, with the last row stating the net increase (decrease) in cash.

As evident from the statement, the company has received \$2100 in income in September. Net income is regarded as a cash inflow from operating activities. “Purchase of trucks” and “Purchase of office equipment” are cash outflows resulting from investing activities, thus these are recorded as negative values on the statement under cash flows from investing activities. Net cash used by investing activities totaled -\$31,500 for September. “Proceeds from sales of capital stock” are categorized as cash inflows from financing activities. Net cash provided by financing activities totaled \$40,000. Adding the “net cash provided by operating activities”, “net cash used by investing activities” and “net cash provided by financing activities”, we get the total change in net cash for the period. In this scenario, we get a net increase in cash:

$$\text{Net increase in cash} = \$3,600 - \$31,500 + \$40,000 = \$12,100$$

Notice, that in this example Pere Courier had to sell stock to finance investing activities of the company. Without proceeds from sales of the capital stock, the company would have a net decrease in cash in September.

### **Real-world example: Canadian Tire’s 2015 fiscal year.**

Let’s take a look at Canadian Tire’s cash flow statement for the 2015 fiscal year:

Canadian Tire Corporation, Ltd.		
Cash Flow Statement		
For the period ended January 2nd 2016		
For the years ended (C\$ in millions)	January 2nd, 2016	January 3rd, 2015
<b>Cash generated from (used for):</b>		
<b>Operating activities</b>		
Net income	\$ 735.9	\$ 639.3
Adjustments for:		
Depreciation of property and equipment and investment property	312.8	279.2
Income tax expense	265.4	238.9
Net finance costs	92.8	108.9
Amortization of intangible assets	111.9	93.1
Changes in fair value of derivative instruments	6.9	-33.9
(Gain) on disposal of property and equipment, investment property, assets held for sale, intangible assets, and lease terminations	-43.9	-9.0
Change in fair value of redeemable financial instruments	-	17.0
Interest paid	-101.4	-122.0
Interest received	8.4	10.4
Income taxes paid	-284.0	-256.5
Other	14.6	25.0
	1119.4	990.4
Change in operating working capital and other	-115.3	-83.5
Change in loans receivable	-25.2	-332.4
<b>Cash generated from operating activities</b>	<b>978.9</b>	<b>574.5</b>
<b>Investing activities</b>		
Additions to property and equipment and investment property	-515.9	-538.6
Additions to intangible assets	-94.7	-150.1
	-610.6	-688.7
Acquisition of short-term investments	-177.4	-431.6
Proceeds from the maturity and disposition of short-term investments	426.6	665.3
Acquisition of long-term investments	-35.0	-155.8
Proceeds from the disposition of long-term investments	-	7.6
Proceeds on disposition of property and equipment, investment property, and assets held for sale	101.5	21.3
Long-term receivables and other assets	0.1	3.1
Other	-4.2	-10.7
	311.6	99.2
<b>Cash (used for) investing activities</b>	<b>-299.0</b>	<b>-589.5</b>
<b>Financing Activities</b>		
Dividends paid	-152.2	-141.4
Distributions paid to non-controlling interests	-53.8	-19.5
	-206.0	-160.9
Net (repayment) issuance of short-term borrowings	-111.2	79.4
Issuance of loans payable	270.1	235.6
Repayment of loans payable	-219.0	-242.4
Issuance of long-term debt	856.1	563.7
Repayment of long-term debt and finance lease liabilities	-588.5	-474.0
Payment of transaction costs related to long-term debt	-6.5	-2.0
	201.0	160.3
Repurchase of share capital	-434.6	-290.6
Proceeds on sale of ownership interests in the Financial Services business	-	500.0
Transaction costs on sale of ownership interests in the Financial Services business	-	-23.2
	-434.6	186.2
Change in deposits	12.5	-97.0
<b>Cash (used for) generated from financing activities</b>	<b>-427.1</b>	<b>88.6</b>
<b>Cash generated in the period</b>	<b>252.8</b>	<b>73.6</b>
<b>Cash and cash equivalents, net of bank indebtedness, beginning of period</b>	<b>647.8</b>	<b>574.2</b>
<b>Cash and cash equivalents, net of bank indebtedness, end of period</b>	<b>\$ 900.6</b>	<b>\$ 647.8</b>

As you've noticed, like the income statement and balance sheet, this statement contains many specific accounting terms. So, once again, reading the explanatory notes that

accompany the statements is useful to get a better understanding of the statement and its entries.

### Examining the Statement of Cash Flows

The Canadian Tire's cash flow statement shows how much cash has been received and how much has been used during 2015 fiscal year. "Cash generated in the period" reflects the net increase (or decrease) in cash over the period. In 2015, Canadian Tire has generated a net increase of \$252.8 million in cash. To determine the company's cash position at the end of the period, we need to add this value to the cash generated in the previous period (cash at the beginning of the period). On this statement this entry is called "cash and cash equivalents, net of bank indebtedness, beginning of period" and is equal to \$647.8 million. Adding the two together, we get the cash at the end of the period, or "cash and cash equivalents, net of bank indebtedness, end of period", \$900.6 million, which is the bottom line of the statement. This value reflects the company cash position as a result of the company's activities throughout the period of the statement.

Recall that cash can be generated, or used for, operating activities, investing activities, and financing activities. So, the cash flow statement is divided into these three sections. Let's go over each category of Canadian Tire's cash flow statement.

**Cash Generated from (Used for) operating activities** is calculated by determining the net income of a business and adjusting it for operating cash inflows and outflows. Net income is found on the income statement and is the business's profit for the period after production costs, operating and non-operating expenses have been accounted for (we obtain it from the income statement). Among all these expenses, there are non-cash expenses, such as depreciation (we will cover depreciation in more detail in section 2.4.2). Although these do not represent a reduction in cash, they make up a portion of the cost of goods sold and are deducted from net income on the income statement. So, to obtain an actual cash amount generated from operating activities, these non-cash expenses are added to the net income to adjust the cash position for the period. Non-operating activities that are accounted for in the investment activities and financial activities sections are also "returned" to net income, so that it reflects the cash flow that was generated specifically from operating activities. These entries are categorized as *adjustments*.

Adjustments are a complex group of items on a cash flow statement. Different businesses will have different items for their adjustments. The adjustments that Canadian Tire has made include: "Depreciation of property and equipment and investment property", "Income tax expense", "Net Finance Costs", "Amortization of intangible assets", "Changes in the fair market value of derivative instruments", "(Gain) on disposal of property and equipment, investment property, assets held for sale, and lease terminations", "Changes in fair value of redeemable financial instruments", "Interest paid", "Interest received", "Income taxes paid" and other adjustments.

To get cash generated from operating activities, the "change in operating working capital" and "change in loans receivable" (accounts receivable) are also accounted for on Canadian Tire's cash flow statement. *Operating working capital*, often referred to as simply working capital, are the liquid assets available to continue and expand operations. Working capital is calculated through the balance sheet and its change in value is added or subtracted

from net income to account for cash and other liquid assets available for operations. We will discuss working capital further in Section 2.2.3. *Loans receivable* represent a customer's obligation to pay back a loan to the company. These obligations represent a part of company's income that is yet to be received, and thus it has to be subtracted from the income to get the actual cash amount.

So, cash generated from operating activities is the sum of net income and adjustments, including changes to working capital and loans receivable, and amounts to \$978.9 million for the 2015 fiscal year.

**Cash Generated from (Used for) Investing Activities** is calculated by adding the cash inflows and outflows from investing activities of the company. Canadian Tire's cash outflows include "additions to property and equipment", "addition to intangible assets", "acquisition of short-term investments", "acquisitions of long-term investments", and "other". Cash inflows include "proceeds from short-term investments", "proceeds from long-term investments", "proceeds on disposition of property and equipment, investment property, and assets held for sale" and "long term receivables and other assets". For the 2015 fiscal year, Canadian Tire's cash used for investing activities totaled \$299 million.

**Cash Generated from (Used for) Financing Activities** is calculated by summing the cash inflows and outflows that result from the business financing its operations. These include the costs of issuing debt, distributing dividends to shareholders, and repurchasing shares. The financing activities included on Canadian Tire's statement of cash flows are "dividends paid", "distributions paid to non-controlling interests", "repayment and issuance of short-term borrowings", "issuance" and "repayment of loans payable", "issuance" and "repayment of long-term debt", "payment of transaction costs related to long-term debt" and others. Cash used for financing activities totaled \$427.1 million.

Adding the "cash generated by operating activities", "cash used for investing activities" and "cash used for financing activities", we get the net cash generated in the 2015 fiscal year:

Cash generated in the period = \$978.9 million – \$299 million – \$427.1 million = \$252.8 million

Notice that we add cash generated from operating activities and subtract cash used for investing and financing activities. This is because in case with cash from operating activities, the cash inflows outweigh cash outflows, while the opposite is true for cash used for investing and financing activities. This is why we say cash *generated from* operating activities, and *cash used for* investing and financing activities.

Thus, as we see, Canadian Tire has a cash surplus from its activities for the 2015 fiscal year, with net cash position at the end of the period being \$900.6 million.

The income statement, the balance sheet, and the statement of cash flows show the results of management's past decisions. They are the end products of the accounting process. These financial statements give a picture of the solvency and profitability of the company. The most recent financial statements for most companies can be found on their websites under "Investor Relations" or some similar heading.



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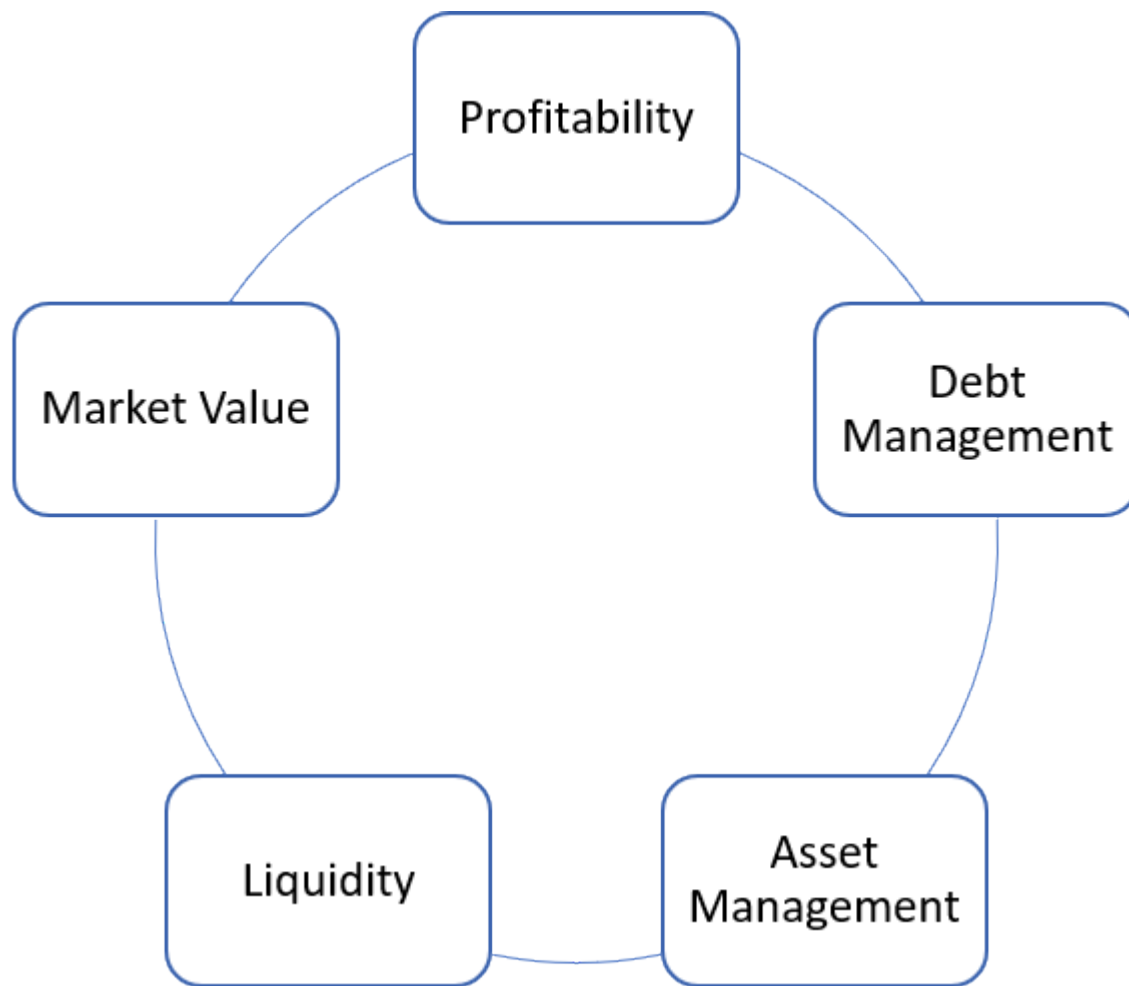
## 2.2 Financial Ratios

Financial statements provide key information about a company's operation. A more in depth data about a company's performance can be revealed by calculating financial ratios using the information from the statements. A *financial ratio* is a measure of the relative magnitude of two selected numerical values. For instance, the *gross profit ratio* expresses the numerical relationship between gross profit and sales. If a company has a gross profit ratio of 0.25, this means that for every \$1 of sales, the company earns \$0.25 on average to cover expenses other than cost of goods sold. Another way of stating this is to say that the gross profit ratio is 25%.

Financial ratios are an effective tool for measuring the financial performance of a company. They provide a common basis for evaluation—for instance, the amount of gross profit generated by each dollar of sales for different companies. Numbers that appear on financial statements need to be evaluated in context. It is their relationship to other numbers and the relative changes of these numbers that provide insight into a company's performance. For this reason, ratios are often compared to industry averages to get a better picture of how the company is doing, or to past years' ratios to determine trends in a company's performance.

There are five major types of financial ratios:

- **profitability ratios** that measure various levels of return on sales, total assets employed, and shareholders' investment;
- **debt management ratios** that measure the financial structure of a company, its amount of relative debt, and its ability to cover interest expense;
- **asset management ratios** that measure the company's inventory and asset profitability and company's ability to collect on accounts receivable;
- **liquidity ratios** that measure the ability of a company to satisfy demands for cash as they arise in the near-term (such as payment of current liabilities); and
- **market value ratios** that measure financial returns to shareholders, and perceptions of the stock market about the company's value.



*Figure 2.5 Financial Ratios*

It is important to note that financial ratios are only one of the tools used in company's performance analysis. They show only one side of the story. For example, a company may have sold hundreds of refrigerators last year and all of the key financial indicators may suggest growth. However, trade publications and consumer reports may indicate the arrival of new technologically advanced refrigerators in the next few years. So, if the company does not have the capacity or necessary equipment to produce these new appliances, the present positive financial indicators may not accurately reflect the likely future of the company.

In this section, we will calculate financial ratios to get some insight into Canadian Tire's performance. The ratios will be obtained based on information from Canadian Tire's income statement and balance sheet for the years 2015 and 2016. While the ratios themselves often provide useful information to draw conclusions about the company's performance, it is usually a good idea to compare the ratios to industry averages. These often act as a benchmark in ratio analysis. There are other techniques used to calculate optimal ratio values for the company, however they are beyond this course. Here, we will focus on main financial indicators and their calculations to give you a better idea about the information they provide.

### 2.2.1 Profitability Ratios

The goal of most companies is to earn profit. To see how well they meet this goal, profitability ratios are commonly used. These ratios are used to determine how efficiently the profits are generated considering company's expenses and assets. In this section, we will cover the following profitability ratios:

- *profit margin ratios,*
- *return on assets ratio (ROA), and*
- *return on shareholder's equity (ROE) ratio.*

Profit margin ratios examine what percentage gross profit, operating profit and net profit constitute in total sale revenue of a company. The ratios that will be covered in this section, respectively, are *gross profit ratio*, *operating profit ratio* and *net profit ratio*.

#### Gross Profit Ratio

The gross profit, or gross margin, is the amount of revenue left to pay operating expenses, non-operating expenses and income taxes after deducting cost of production. The *gross profit ratio* indicates the percentage of sales revenue that constitutes gross profit. In other words, it shows how much gross profit is in every 1\$ of company's total sales. The ratio is calculated as:

$$\text{Gross Profit Ratio} = \text{Gross Profit} / \text{Sales Revenue} \quad (2.7)$$

The higher the value of the ratio, the better the production expenses strategy the company has. When analyzing dynamics of the ratio, we can see whether a company is making more profit relative to production costs or not.

Using the information from Canadian Tire's income statement we can calculate the gross profit ratio for 2016 and 2015:

<i>In Millions of Dollars</i>		<b>2016</b>	<b>2015</b>
<b>Gross Profits</b>	<b>a</b>	<b>\$ 4,135.3</b>	<b>\$ 4,046.0</b>
<b>Net Sales</b>	<b>b</b>	<b>\$12,279.6</b>	<b>\$12,462.9</b>
<b>Gross Profit Ratio</b>	<b>a/b</b>	<b>0.34</b>	<b>0.32</b>

\*Note that the figures in statements are reported in millions of dollars (000,000s).

So, the gross profit ratio is 34% in 2015 and 32% in 2014. In other words, for each dollar of sales Canadian Tire has \$0.34 of gross profit left to cover operating, non-operating expenses and income taxes in 2015, compared to \$0.32 in 2014. The ratio has increased by 6% in 2015. This positive change implies that the company improved the company's production by decreasing production costs, or by changing company's pricing policy. So, in 2015, the company operated more profitably.

#### Operating Profit Ratio

The operating profit is the profit left after production costs and operating expenses have been deducted. It is used to finance non-operating expenses and income taxes. The *operating profit ratio* shows what percentage of sales is the operating profit. That is, it shows how much operating profit is in every 1\$ of company's total sales. It is calculated as:

$$\text{Operating Profit Ratio} = \text{Operating Profit} / \text{Sales Revenue} \quad (2.8)$$

Similarly to the gross profit ratio, the higher the value of the operating profit ratio, the better. Canadian Tire's operating profit ratio for the 2015 and 2014 fiscal years is calculated as follows:

As we see, the operating profit ratio is 8.5% in 2015 and 8% in 2014. For every dollar of Canada Tire's sales, the company has \$0.085 of operating profit in 2016 and \$0.08 in 2014. The ratio has increased by 6%. This change implies that Canadian Tire has decreased its operating costs per 1\$ of total sales, so now it operates more profitably.

### Net Profit Ratio

Net profit represents a company's income after production costs, operating and non-operating expenses and income taxes have been accounted for. So, the *net profit ratio* is the percentage of sales revenue retained by the company after deducting all the expenses and taxes. This ratio is commonly used when comparing companies in the same industry. This ratio is obtained by dividing net income by total sales revenue of the company:

$$\text{Net Profit Ratio} = \text{Net Income} / \text{Sales Revenue} \quad (2.9)$$

The higher the value of the ratio, the more efficiently the company's expenses are. When analyzing dynamics of the ratio, we can see whether a company is making more profit relative to its expenses or not.

Canadian Tire's net profit ratio for 2015 and 2014 are calculated as follows:

<i>In Millions of Dollars</i>		Ending <u>January,</u> 2016	Ending <u>January,</u> 2015
Net Income	a	\$ 735.9	\$ 639.3
Net Sales	b	\$12,279.6	\$12,462.9
Net Profit Ratio	a/b	0.06	0.05

So, the net profit ratio was 6% in 2015 and 5% in 2014. This indicates that for each \$1 of sales, Canadian Tire earned \$0.06 of net income in 2015 and \$0.05 in 2014. This ratio has increased by 20% in 2015, which shows that Canadian Tire's profitability has increased, while the relative expenses incurred to generate these profits decreased.

Recall that revenues are generated using company's assets. The financial strength and success of a company

depends on how profitably these assets are used. *Return on assets (ROA) ratio* determines how successful a company is at using its assets to generate profit.

### Return on Assets (ROA) Ratio

Return on assets ratio examines whether a company's sales are adequate in relation to its assets. The ratio tells us how much money is earned by each dollar invested in assets. The ratio is calculated as:

$$\text{Return on Assets} = \text{Net Income} / \text{Average Total Assets} \quad (2.10)$$

Note that in the denominator average total assets are used. Recall that the information on the balance sheet reflects a *state*, rather than a *flow*, like the income and cash flow statements. So, when using numbers from the balance sheet and income statement in ratios with a goal of estimating a ratio over a period of time (e.g. a year), the numbers from the balance sheet need to be averaged over the period to get a more accurate representation of company's financials. This is because company's total assets, liabilities and equity can vary throughout a year (companies sell and purchase equipment, acquire and pay off debts, issue stock etc.), and this must be accounted for when calculating ratios for a period of time, rather than a specific point in time.

This is why when calculating the ROA, we use average total assets. We calculate them by averaging the value at the beginning of the year (end of 2014 when calculating ROA for 2015), and end of the year (end of 2015). So, for 2015, the average total assets were:  $(\$14,553.2 + \$14,987.8)/2 = \$14,770.5$  million.

Thus, the ROA for 2015 is:

In Million of Dollars		Ending <u>January</u> , 2016
Net Income	a	\$735.9
Average Total Assets	b	\$14,770.5
ROA	a/b	0.05

As we see, the ROA is 5% in 2015. That is, 1\$ of assets generated \$0.05 of net income in 2015. To get a better idea of whether this is a good value, it is generally advisable to compare the ratios with the industry averages. If the ROA is higher than the industry average, or is increasing, this implies that the assets are used efficiently. Usually, an upward trend for the ROA suggests that assets are being used more profitably, while a downward trend indicates that assets are being used less profitably. However, note that after investing in some assets, the pay off might not appear in the same period, so a drop in the ROA in the current period might be evident. This is due to an increase in total assets on the balance sheet, which has not been accompanied by a corresponding increase in production and, thus, income. In such cases, production profitability is likely to increase in subsequent periods, and so a drop in the ROA for the period is associated with an investment in assets, rather than production inefficiency.

- Final profitability ratio in this section reflects how much income is earned per dollar of shareholder's investment. This ratio is especially important for shareholders as it provides information on how much money a company has made using the money the shareholders have invested into it.

### Return on Shareholder's Equity Ratio (ROE)

The *return on shareholders' equity ratio (ROE)*, commonly referred to as return on equity, measures how much net income was earned per dollar of equity. The ratio is calculated as:

$$\text{Return on Shareholder Equity} = \text{Net Income} / \text{Average Shareholder's Equity} \quad (2.11)$$

Similarly to the ROA calculation, we need to average shareholder's equity, since we obtain this number from the balance sheet. We do this by averaging the shareholder's equity at the beginning of period (end of 2014) and end of period (end of 2015), to get average shareholder's equity for 2015:  $(\$5,630.8 + \$5,789.7)/2 = \$5,710.25$

The 2015 returns on shareholders' equity ratio for Canadian Tire is calculated as follows:

In Million of Dollars		Ending <u>January</u> , 2016
Net Income	a	\$735.9
Average Shareholder's Equity	b	\$5,710.25
ROE	a/b	0.13

So, in 2015 the ROE was 13%. This means, that \$0.13 net income was earned for every \$1 of Canadian Tire's equity. To get a better understanding of whether or not it is a good value, the ratio should be compared to industry averages. If, for example, the industry average was 8%, then we could conclude that Canadian Tire performed better than the average company in the industry.

### 2.2.2 Debt Management Ratios

A company needs assets to be able to produce goods and services. To acquire assets, a company must raise necessary funds. This can be done through raising equity by issuing and selling stock, or by getting debt. Usually, companies will use both channels, thus raising some money through equity, and some through loans. There are three ratios to consider when analyzing company's finances, and how it manages its debts: *debt to equity ratio*, *debt to assets ratio* and *times-interest-earned ratio*.

### Debt to Equity Ratio

The ratio of company's total liabilities to total equity is called the *debt to equity ratio*:

$$\text{Debt to Equity} = \text{Total Liabilities} / \text{Total Equity} \quad (2.12)$$

This ratio is used to determine how much liabilities the company has per dollar of equity. Using the information from the balance sheet, we calculate the debt to equity ratio as follows:

<i>In Million of Dollars</i>		As of <u>January</u> , 2016	As of <u>January</u> , 2015
Total Liabilities	a	\$9,198.10	\$8,922.4
Shareholder's Equity	b	\$5,789.70	\$5,630.80
Debt to Equity Ratio	a/b	1.59	1.58

Note that here we do not need to average liabilities and equity, because we are interested in the financial *state* of the company, rather than progress or change over a period of time.

So, Canadian Tire had \$1.59 of liabilities for each dollar of shareholders' equity as of January 2nd 2016, and \$1.58 per dollar of as of January 2nd, 2015. The proportion of debt financing had increased, which means that the company acquired relatively more debt than equity.

On one hand, management's increasing reliance on creditor financing has its advantages. Issuing additional shares might require existing shareholders to give up some of their control of the company, thus diluting the shareholders' ownership interest in the company. This can be omitted when acquiring debt instead. Creditor financing may also be more financially attractive. Usually, loans are "cheaper" compared to raising money through selling shares in the long run. By getting a loan, a company has a specific amount plus interest that needs to be repaid, thus a creditor has no direct claim on profits of the company. Moreover, the bureaucratic hassle of getting a loan is less complicated.

On the other hand, management's increasing reliance on creditor financing poses certain risks. Firstly, one must understand that the loan must be repaid. When a company acquires a large amount of loans, it might face difficulties to pay them off. This has a negative impact on credibility of the company. Moreover, companies that have large amounts of debts must spend a lot of money on interest and loan payments, which might significantly decrease company's net income and impede company's growth until a considerable amount of the loan is paid off.

It is also important to note that when creditors decide whether to lend money to a company, they study its total debt load and ability to meet loan payments in the future. So, the higher the debt to equity ratio, the riskier it is for the creditor to borrow money to the company, and the harder it will be for the company to acquire new loans. Thus, an increasing debt to shareholders' equity ratio could hinder company's borrowing capacity in the future.

Although there is no single appropriate debt to shareholders' equity ratio, there are techniques for estimating the optimum balance. These are beyond the scope of the course and for now it is sufficient to note that, for Canadian Tire, the debt to shareholders' equity ratio has increased over the last year. A continuing positive trend into the future is generally viewed unfavourably because of the risk associated with increased debt financing.

### Debt to Assets Ratio

The *debt to assets ratio* describes the relationship between the total liabilities and total assets of the company. It can be calculated as follows:

$$\text{Debt to Assets} = \text{Total Liabilities} / \text{Total Assets} \quad (2.13)$$

The debt to assets ratio shows what percentage of assets is company's debt. A ratio of 1 would indicate that all of the assets have been financed through debt. If the ratio is below 1, this means that assets of the company outweigh company's debt, and can therefore be used to finance debt if the need arises and the company cannot generate enough profit to repay its debts. If the ratio is higher than 1, this means a company has more debt than it does assets. As a general rule, the higher the debt ratio, the more difficult it may be for a company to find new financing.

Using the information from Canadian Tire's balance sheet we calculate the debt to assets ratio as follows:

<i>In Million of Dollars</i>		As of <u>January</u> , 2016	As of <u>January</u> , 2015
Total Liabilities	a	\$9,198.10	\$8,922.4
Total Assets	b	\$14,987.80	\$14,533.20
Debt to Assets Ratio	a/b	0.614	0.614

As we see, Canadian Tire's debt to asset ratio at the end of both years is 61%. This means that the company's debt is \$0.61 per dollar of its total assets. The ratio remained constant.

### **Times Interest Earned Ratio**

Creditors evaluate not just the company's financial position, but also a company's financial performance. They do so in order to determine whether it will be able to pay interest on borrowed funds and repay debt when it comes due. One ratio that is commonly used to analyze company's financial performance is the *times interest earned ratio*. This ratio shows how many times income from operations could be used to pay the interest expense on the loan. The larger the ratio, the better creditors are protected. If operating income declines, a company might not be able to pay interest on the loan. The ratio is calculated using the following formula:

$$\text{Times Interest Earned} = \text{Operating Income} / \text{Interest Expense} \quad (2.14)$$

Here, operating income is used, since it is used to cover non-operating expenses. Recall that expenses associated with acquiring and servicing debt belong to non-operating expenses. Using the information from Canadian Tire's income statement, the times interest earned ratio is:



<i>In Million of Dollars</i>		Ending <u>January</u> , 2016	Ending <u>January</u> , 2015
Operating Income	a	\$1,039.20	\$993.1
Interest Expense (Net Finance Costs)	b	\$92.80	\$108.90
Times Interest Earned Ratio	a/b	11.20	9.12

So, in 2015 fiscal year the times earned interest ratio was 1120%, and 912% in 2014 fiscal year. Both values are high, which is a good sign for potential creditors and the company. Canadian Tire's operating profit is sufficient to cover interest owed by a company. Moreover, the ratio has increased in 2015, showing that the company generates even more profit for every dollar of interest expense compared to previous year.

### 2.2.3 Liquidity Ratios

Liquidity is the ability of a company to satisfy demands for cash as they arise in the near-term future (such as payment of current liabilities). One term that is often used when assessing company's liquidity is working capital.

**Working capital** is the difference between a company's current assets and current liabilities at a point in time:

$$\text{Working Capital} = \text{Current Assets} - \text{Current Liabilities} \quad (2.15)$$

The larger the working capital, the more assets can be converted to cash to pay off the current liabilities, and the more liquid a company is. The need for working capital arises due to company's need for immediate cash to ensure that its operations are running smoothly. For example, think of Walmart's need for cash in their registers when they open a new store. If there was no cash in registers, it would be difficult to run the store without holding up customers until enough change can be gathered. Working capital is also required to fund inventory and accounts receivable.

We can calculate the working capital for Canadian Tire using its balance sheet:

<i>In Millions of Dollars</i>		As of January, 2016	As of January, 2015
<b>Current Assets</b>			
Cash and cash equivalents		\$ 900.6	\$ 662.1
Short-term investments		96.1	289.1
Trade and other receivables		915.0	880.2
Loans receivable		4,875.5	4,905.5
Merchandise inventories		1,764.5	1,623.8
Income taxes recoverable		42.2	31.9
Prepaid expenses and deposits		96.1	104.5
Assets classified as held for sale		2.3	13.1
Total Current Assets (a)		<u>8,692.3</u>	<u>8,510.2</u>
<b>Current Liabilities</b>			
Bank indebtedness		\$ -	\$ 14
Deposits		880.7	950.7
Trade and other payables		1957.1	1961.2
Provisions		216.1	206.0
Short-term investment borrowings		88.6	199.8
Loans payable		655.5	604.4
Income taxes payable		61.5	54.9
Current portion of long-term debt		24.3	587.5
Total Current Liabilities (b)		<u>3,883.8</u>	<u>4,578.8</u>
Working Capital (a-b)		<u>\$ 4,808.5</u>	<u>\$ 3,931.4</u>

In the schedule above, working capital amounts to \$4,808.5 million as of January 2nd, 2016, and \$3,931.4 million as of January 2nd, 2015. As we see, the working capital had increased by \$877.1 million (\$4,808.5 – \$3,931.4). So, Canadian Tire's liquidity has increased and the company has more working capital available as of January 2nd, 2016. This means that the company has more cash to run its business and pay its short-term liabilities as they become due.

In addition to calculating an absolute amount of working capital as a measure of liquidity, ratio analysis is also commonly used. The two ratios that are frequently calculated are the *current ratio* and the *quick ratio*.

### Current Ratio

Is Canadian Tire able to repay short-term creditors? The *current ratio* can help answer this question. The ratio shows the relationship between company's current assets and current liabilities and is calculated as:

$$\text{Current Ratio} = \text{Current Assets} / \text{Current Liabilities} \quad (2.16)$$

The ratio indicates how many times a company's current assets can be used to pay its current liabilities. If the ratio is equal to 1, this means that current liabilities can be paid off with current assets, however acquiring any more current liabilities would result in a company not being able to meet its obligations in the current period. If the ratio is less than one, the company is considered insolvent short-term, since it is unable to repay current liabilities with the current assets it owns. Companies usually aim to have this ratio above 1. The higher the ratio, the better the ability the company has to pay off the current liabilities, and the more protected the company and its creditors are. However, when the ratio is too high, it could imply that there is too much working capital, that might not be needed. Thus, it could be used inefficiently. Recall the Walmart example. There is no need to have extreme amounts of cash in their registers. They need just enough cash to keep the operations running smoothly without unnecessary hold ups. All the extra cash would just be lying pointlessly in the registers, even though it could be used to create profit. When analyzing the dynamics of the ratio, we can see whether the company's liabilities are rising faster than their assets.

Using the information from Canadian Tire's balance sheet, we calculate the current ratio as follows:

<i>In Millions of Dollars</i>		As of <u>January</u> , 2016	As of <u>January</u> , 2015
Current Assets	a	\$ 8,692.3	\$ 8,510.2
Current Liabilities	b	\$ 3,883.8	\$ 4,578.8
Current Ratio	a/b	2.24	1.86

As we see, as of January 2nd ,2016, the current ratio was equal to 224%, and as of January 2nd, 2015, the current ratio was 186%. This means that as of January 2nd ,2016, there were \$2.24 of current assets per each \$1 of current liabilities, compared to \$1.86 as of January 2nd, 2015. So, Canadian Tire has increased its liquidity in 2015. To gauge whether this ratio is high enough, it is usually compare to industry's average ratios. Information on these is available in various trade publications and business analysts' websites that assemble financial ratio information for a wide range of industries.

### Quick (Acid-Test) Ratio

Another common liquidity ratio is a *quick ratio*, which is also called an acid-test ratio. To calculate this ratio, current assets are separated into quick current assets and non-quick current assets. The idea is that current assets are used short-term, but even among the current assets there are those that can be used almost immediately (quick assets) and those, that might need some time before they can be converted into cash or used as payment (non-quick assets). Quick current assets include cash, accounts receivables, and marketable securities because they are easily converted to cash and can be used as payment almost immediately. Non-quick current assets include inventories

and pre-paid expenses. Pre-paid expenses cannot be redeemed for cash value but are usually used up within a year. Inventories are usually sold within a year. However, if a company cannot wait to find a buyer who will pay full value for the asset, they may have to offer a discount.

The acid-test ratio is calculated as:

$$\text{Quick Ratio} = \text{Quick Current Assets} / \text{Current Liabilities} \quad (2.17)$$

The ratio shows if the company would be able to pay off its current liabilities if they become due immediately. It indicates how many quick current assets dollars there are for each dollar of current liabilities. The higher the value of the ratio, the more protected the company and creditors are. However, just like in case with the current ratio, high values of quick ratio might be associated with capital that is not used efficiently (it could be used to create more profit). The parallel is the same as in the Walmart example brought up earlier in this section.

So, what is an adequate quick ratio? It is generally considered that the ratio of 1 is adequate to ensure that a company will be able to pay its current obligations. However, this is a fairly arbitrary guideline and is not appropriate in all situations. A ratio lower than 1 can often be found in successful companies. Usually, companies' management decides on the level of quick assets to target. This decision is dependent on the company's strategy, nature of business and company's perspectives.

To obtain the ratio, we first need to calculate the quick assets. We get the information to calculate the ratio and quick assets from Canadian Tire's balance sheet. We get:

		As of January, 2016	As of January, 2015
<i>In Millions of Dollars</i>			
<b>Quick Current Assets</b>			
Cash and cash equivalents		\$ 900.6	\$ 662.1
Short-term investments		96.1	289.1
Trade and other receivables		915.0	880.2
Loans receivable		<u>4,875.5</u>	<u>4,905.5</u>
Total Quick Current Assets	a	<u>\$ 6,787.2</u>	<u>\$ 6,736.9</u>
Current Liabilities	b	<u>3,883.8</u>	<u>4,578.8</u>
Quick Ratio	a/b	1.75	1.47

As of January 2nd, 2016, the quick ratio was 175% compared to 147% as of January 2nd, 2015. This means that as of January 2nd, 2016, for every 1\$ of current liabilities there was \$1.75 of quick assets, compared to \$1.47 as of January 2nd, 2015. In Canadian Tire's case, the ratio values were more than 1, so the company has enough quick assets to pay off its current liabilities. Although the ratio seems high, it may be reasonable. Again, this depends

on the company's strategy decided by management. High values of the quick ratio provide Canadian Tire with flexibility in future planning. From creditor's perspective, an increasing quick ratio is a good sign. It implies that a company has a better financial ability to pay off its short-term obligations and a reduced risk of insolvency.

#### 2.2.4 Asset Management Ratios

Asset management ratios allow to analyze how efficiently the assets are used by a company. These ratios include: *inventory turnover*, *days sales outstanding* and *total asset turnover*.

##### **Inventory Turnover Ratio**

*Inventory turnover ratio* is used to determine how many times inventory is sold and restocked each year. It is calculated as follows:

$$\text{Inventory Turnover Ratio} = \text{Cost of Goods Sold} / \text{Average Inventory} \quad (2.18)$$

Other sources might use sales in the numerator, rather than the cost of goods sold. This approach is also valid, however it might overestimate the ratio. This is because sales include a markup, while the cost of goods sold represents just the costs of the products. So, dividing the cost of goods sold by the average inventory yields a more precise inventory turnover ratio.

The ratio is usually compared to industry averages. As a general rule, management does not want the ratio to be too high or too low. A high turnover implies big sales, and that the company might face stock outs. If the inventory turnover is too low, this means a company has excessive inventory and the sales are weak.

An increasing turnover ratio is usually a sign of good inventory management. It indicates that the average amount of assets tied up in inventory is declining. With lower inventory levels, inventory-related expenses such as rent and insurance are lower because less storage space is often required. However, as mentioned above, lower inventory levels can have negative consequences since items that customers want to purchase may not be in inventory, resulting in lost sales. Having said this, decreasing inventory turnover is usually a sign of poor inventory management because an excessive investment in inventory ties up cash that could be used for other purposes. Increasing levels may indicate that inventory is becoming obsolete (for example in an electronics company) or deteriorating (for example in a company that sells perishable groceries). Obsolete or deteriorating inventories may be unsalable.

Also, note that when comparing the ratios, it is imperative to know how the ratio was calculated (whether sales or cost of goods sold was used in calculations).

Using Canadian Tire's income statement and balance sheet we calculate the inventory turnover ratio as follows:

<i>In Millions of Dollars</i>		Ending <u>January</u> , 2016
Cost of Goods Sold	a	\$ 8,144.3
Average Inventory		
[(opening balance + closing balance)/2]	b	<u>1,694.15</u>
Inventory Turnover Ratio	a/b	4.81

Note that the average inventory is used to calculate the ratio, since we use values from both income statement and balance sheet and are calculating the ratio for a period of time, rather than for a specific point in time. The average inventory is  $(\$1,764.5 + \$1,623.8)/2 = \$1,694.15$  million. The inventory turnover ratio indicates that Canadian Tire turned its inventory over 4.81 times in 2015 fiscal year. That is, Canadian Tire sold and restocked its inventory 4.81 times in the year. Comparing the ratio with industry averages and further analysis of Canadian Tire's management strategy would provide more insight and is usually advisable when performing a comprehensive financial analysis of the company.

### Days Sales Outstanding

The *day's sales outstanding* ratio, also called the *average collection period* of accounts receivable, is used to calculate how long it takes on average for the company to collect its payments from customers after the purchase has been made. In other words, it shows the average term of accounts receivables. The ratio is calculated the following way:

$$\text{Days Sales Outstanding} = \text{Receivables} / \text{Average Sales per Day} \quad (2.19)$$

Here, the average sales per day are the total sales of the company divided by 365 days.

The lower the ratio, the lower the average time needed to collect customer's owing money. That is, having shorter average collection period of account receivable means that the company gets paid for its goods and services faster, and is thus more liquid (accounts receivables are faster converted into cash). The higher the ratio, the more time it takes for the customers to pay their debts. High ratios are usually a sign of poor accounts receivable management. The longer accounts receivable remain outstanding, the greater the probability they never will be collected. Thus, generally, companies aim to have shorter average collection period of accounts receivable.

We calculate the ratio using Canadian Tire's income statement and balance sheet.

Note that for this calculation we used information from both the balance sheet and income statement and the ratio is calculated to reflect a period rather than state. Therefore, we need to average accounts receivable (loans receivable on the balance sheet), by adding the values of the accounts receivable at the beginning of the year (as of January 3rd, 2015) and at the end of 2015 (as of January 2nd, 2016), then dividing it by 2:  $(\$4,905.5 + \$4,875.5)/2 = \$4,890.5$  million. The average sales per day in 2015 were:  $\$12,279.6/365 = \$33.64$  million.

<i>In Million of Dollars</i>		Ending <u>January</u> , 2016
Accounts Receivable	a	\$4,890.5
Average Sales per Day	b	\$33.64
Days Sales Outstanding	a/b	145.38

So, as we see, Canadian Tire's average collection period of accounts receivable is 145.38 days. It is pretty high, since the ratio implies that on average the customers need 145 days to pay their debts to the company. In this case, it is usually advisable to study the structure of accounts receivable. Is the ratio high because of a few clients that owe big amounts or is it because it takes most customers almost 5 months to pay what they owe to the company? If the former is the case, the management will have to change their policy to avoid customers that are less credit-worthy in the future. If the problem is the majority of customers not paying in general, the company might want to create incentives to motivate their customers to pay faster, such as possible discounts or access to special promotions. On the other hand, if the ratio is high because the company offers credit on big purchases, sales of which would otherwise stumble without customer's access to credit, then this ratio may be very reasonable, even if it might seem high at first glance. Whichever the case, this example illustrates why just having the value of the ratio may not be enough to accurately assess company's operations and more information is needed to arrive to reliable conclusions.

### Total Assets Turnover

*Total assets turnover* ratio shows the relationship between the dollar volume of sales and the average total assets used in the business. It is calculated as follows:

$$\text{Total Asset Turnover} = \text{Sales} / \text{Total Assets} \quad (2.20)$$

This ratio measures the efficiency with which a company uses its assets to generate sales. The larger the total assets turnover, the larger the income on each dollar invested in the assets of the business.

Using the information from Canadian Tire's balance sheet and income statement we calculate the ratios as follows:

<i>In Million of Dollars</i>		Ending <u>January</u> , 2016
Sales	a	\$12,279.6
Average Total Assets	b	\$14,770.5
Total Assets Turnover	a/b	0.83

Note that again, in the formula we use average total assets for the period since we are calculating the ratio for a period of time and using information from the balance sheet, which contains on the state of the company.

So, the total assets turnover ratio for Canadian Tire is equal to 0.83. This means, that each dollar invested in total assets of the company generates \$0.83 in sales in 2015.

### 2.2.5 Market Value Ratios

As we have already mentioned, companies often finance their operations by acquiring money through selling their stock. Investors purchase the stock to get a return on the money they invest. But how do we know if buying a certain company's stock is a good investment? How can one compare investing in one company's stock to another company's stock? These questions can be answered by using the so-called **market value ratios**. The goal of these ratios is to provide information on financial returns on company's stock to investors. The three market value ratios that we will cover are: *earnings per share*, *price-earnings ratio* and *dividend yield*.

#### Earnings per Share (EPS)

Many stockholders look at the *earnings per share (EPS)* ratio when analyzing their investments options. This ratio reflects shareholders' returns on a per-share basis. The higher the ratio, the more profit is earned on each share. To get this ratio, the net income earned in a year is divided by the number of shares to establish how much return has been earned on each share:

$$\text{Earnings Per Share} = \text{Net Income} / \text{Number of Shares Outstanding} \quad (2.21)$$

From Canadian Tire's income statement, we can see that there were 78,960,025 shares at the end of 2015 and 76,151,321 shares at the end of 2016. The net income in 2016 was \$659.4 million and \$604.0 million in 2015. So, using formula 2.7, the EPS values for 2015 and 2016 are:

<i>In Millions of Dollars</i>		Ending <u>January</u> , 2016	Ending <u>January</u> , 2015
Net Income	a	\$ 659.4	\$ 604.0
Number of Outstanding Shares	b	76,151,321	78,960,025
Earnings Per Share	a/b	\$ 8.66	\$ 7.65

\*Note: We must use the net income attributable to shareholders in order to accurately calculate the earnings per share instead of total net income because part of it is from attributions from non-controlling interests.

In 2015, Canadian Tire's EPS was \$7.65. In 2016, the EPS has increased to \$8.66 (a 13.2% increase) due to an increase in net income and a reduction of the number of outstanding shares. In 2016, Canadian Tire bought back common shares, reducing the common shares and, therefore, having a positive effect on EPS. So, as we see, in 2016 each share earns more compared to 2015.

#### Price-earnings (P/E) Ratio

The *price-earnings ratio (P/E)* reflects the shares' value (market price) in terms of its earnings. To get this ratio, we divide the market price per share by EPS:



$$\text{Price-Earnings Ratio} = \text{Market Price per Share} / \text{Earnings Per Share} \quad (2.22)$$

The market price of a share reflects the investors' opinions about a company's future value compared to alternative investments. For example, if it is expected that the company will become more profitable in the future due to its technological advancements etc., investors become more interested in buying this company's shares, since this implies better financial gains for them. This drives the prices of the shares up. By dividing the market price by the EPS, we get how much investors are willing to pay for \$1 of net income generated by the company. Thus, this ratio is used as an indicator of the market's expectation of a company's future performance. Higher values of the price-earnings ratio imply investor's growing expectation.

For example, assume Corporation A has a current market value of \$15 per share and an EPS of \$1 per share. So, its P/E ratio is 15. If Corporation B has a market value of \$4 per share and an EPS of \$0.50 per share, its P/E ratio is 8. This means that the stock market expects Corporation A to earn relatively more in the future than Corporation B. For every \$1 of net income currently generated by Corporation A, investors are willing to invest \$15. In comparison, for every \$1 of net income generated by Corporation B, investors are willing to invest only \$8. Investors therefore perceive shares of Corporation A as more valuable.

Let's go back to Canadian Tire. The company's stock was priced at \$122.22 per share at the beginning of 2015 and \$118.16 at the beginning of 2016. So, using formula 2.8, we can calculate its P/E ratio for 2015 and 2016 follows:

<i>In Millions of Dollars</i>		Ending <u>January</u> , 2016	Ending <u>January</u> , 2015
Market Price Per Share	a	\$ 118.16	\$ 122.22
Earnings Per Share	b	\$ 8.66	\$ 7.65
Price-Earnings Ratio	a/b	13.65	15.98

As we see, Canadian Tire's Price-Earnings ratio has decreased between 2015 and 2016 (it dropped by 14.6%). This indicates that investors are losing confidence in Canadian Tire's ability to earn income. The ratio indicates that investors are now only willing to pay \$13.65 for every \$1 of net income Canadian Tire earns, compared to \$15.98 in the previous year.

### *Dividend Yield*

Many investors' primary objective is to maximize dividend revenue from share investments. In these cases, investors are interested in information about the earnings available for distribution to shareholders and the actual amount of cash paid out as dividends. The dividends amount is then compared to the price of the share to determine how lucrative it is. This ratio is called the *dividend yield* and is calculated as follows:

$$\text{Dividend Yield} = \text{Dividends per Share} / \text{Market Price per Share} \quad (2.23)$$

This ratio indicates how large a return from an investment in a company's shares is. The higher the dividend yield, the more money the owner of company's shares makes.

The relevant information for Canadian Tire over the last two years is shown in the financial statements, as follows:

<i>In Millions of Dollars</i>		Ending <u>January</u> , 2016	Ending <u>January</u> , 2015
Dividends Declared	a	\$ 152.2	\$ 141.4
Outstanding Shares	b	76,151,321	78,960,025
Dividends Per Share	a/b	\$ 2.00	\$ 1.79

The dividend yield ratio is therefore:

<i>In Millions of Dollars</i>		Ending <u>January</u> , 2016	Ending <u>January</u> , 2015
Dividends Per Share	a	\$ 2.00	\$ 1.79
Market Price Per Share	b	\$ 118.16	\$ 122.22
Dividend Yield	a/b	1.69%	1.47%

As we see, Canadian Tire's dividend yield ratio increased from 2015 to 2016. In 2015, investors received \$1.47 for every \$100 invested in shares. By 2016, this increased to \$1.69 for every \$100 invested. Investors seeking dividends would view this as a positive change as they are receiving more dividends in 2016 compared to 2015.

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## 2.3 Forecasting and Financial Statements

### 2.3 Forecasting and Financial Statements

Forecasting is an important aspect of operating a business. It is essential for a company's strategic planning, management, and organization. A financial forecast is an estimate of future financial outcomes of a company. Businesses use forecasts to predict how much they will need to spend on expenses, such as materials, or how much income will be generated. Forecasting can be simple for items that remain consistent in value and quantity. For example, rent is consistent over the short-term and grows moderately in the long term. However, when the values can change significantly, forecasting can be an arduous task. As a general rule, forecasting becomes more complicated as variability increases and more elements have to be accounted for.

Using historical internal accounting and sales data, in addition to external market and economic indicators, a financial forecast is created to reflect a company's possible financial outcomes over a specific time period—which is usually one year. Generally, the main goal of a financial forecast is to predict revenue. Once a company's revenue is estimated, future costs are then predicted. Fixed costs generally remain constant or are subject to little change. Variable costs are usually estimated based on expected sales. After sales and expenses are predicted, the company can then estimate net income for the period.

Unlike a financial plan or a budget, a financial forecast doesn't have to be used as a planning document. It is usually done in order to give company's management and investors an idea of a company's possible performance, upon which the management can decide on the course of actions for the company. Moreover, when forecasting, companies usually estimate several scenarios. This way if certain trends do not continue into the future (for example, due to competitor actions, or unexpected economic changes), a company is less likely to be caught by surprise and can navigate its operations more successfully.

Companies approach forecasting in many different ways. Some companies pay marketing agencies that study industry trends to obtain the recent trend data. Many organizations hire market research consultants to obtain and review industry data to predict consumer demand. Many companies employ economists to develop financial models to project sales. While financial modeling is typically used by big companies, smaller companies with a simpler revenue and expense structure often base their estimates of possible future sales on past trends and customer demand expectations.

#### *Example 2.5*

Suppose a local ice cream company wants to forecast their ice cream sales for summer. In spring, their sales totaled \$26,000 (the company sold 5200 ice cream cones for \$5 each). From previous years, the management has noticed a pattern in sales increasing on average by 40% in summer. So, if the sales in spring were \$26,000, the management expects to make approximately  $\$26,000 \times 1.4 = \$36,400$ , or 7280 ice cream cones for \$5 each. Having an estimate of sales for summer, a company now can forecast its expenses. Fixed expenses, such as rent, will remain the same. Variable costs, such as ingredients and labour to produce ice cream, will have to increase proportionally to the amount of ice cream cones that are expected to be sold in summer. After calculating the expenses, the company can forecast its income by deducting all the estimated expenses from the estimated sales revenue.

Note that forecasting depends not only on the size of the company and its revenues and expenses structure, but also on the type of goods sold. For example, if a company sells ice cream, previous data on company's sales is the main determinant of consumer's demand. If, on the other hand, the company sells luxury cars, to forecast the demand for the products on top of its sales data the company needs to take into account the economic situation in the country, household's income levels, competitor's strategies, industry trends and so on.

In example 2.5, management expects a 40% increase in sales in summer after studying the local demand for ice cream cones. Without knowing the demand, it would be impossible to project company's sales. The better and the more accurate the demand forecast is, the more precise the sales and, correspondingly expenses forecast will be. Without an accurate demand estimate, a company may find itself with an excess of unused products and services if the demand was overstated or unable to serve the customers and lose on potential revenues if the demand was understated. Producing excess products and services is costly, since they could be employed in different ways to produce revenues. For example, they could be invested. On the other hand, when a company's set production targets are too low, the company faces missed opportunities for profit. Moreover, not being able to meet the demand for products and services is likely to discourage potential customers from using the company's products or services in the future.

To a limited extent, a company may be able to alter future demand to be more in line with its capacity because it has control over some determinants of demand, like pricing, promotion, and location. If the business is surprised by demand levels that are higher or lower than expected, these market strategy elements can be adjusted to either stimulate or diminish demand to conform to its production capabilities. For example, if a company that manufactures clothing has overestimated its demand, it can increase its sales by decreasing prices on clothing, or having big sale events. However, even if the company will manage to sell the clothing they initially planned to based on their forecasting, this was done by losing part of their revenues, since they had to decrease their prices to increase the demand. Furthermore, companies can alter demand only to a certain extent, as most businesses have limited ability to affect it.

Forecasts are made in advance. This enables the companies to have an adequately sized operation, sufficient staff in terms of size and training, and obtain any necessary resources for production. These capabilities are usually not possible to achieve overnight. For some goods, production is a process that takes significant time from initiation to completion, such as constructing apartments or office space that will be leased to customers. Even businesses that provide products or services "made to order," where most of the direct organization or production activities

occur after a purchase is made, usually need to have supplies, trained labor, and management structures in place in advance of the order to be in a position to negotiate a sale.

### 2.3.1 *Pro Forma Financial Statements*

After using forecasting tools to predict sales, expenses and income for a period, companies often create financial statements with this information to study forecasted scenarios in more detail. These statements are called **pro forma financial statements**.

Pro forma statements summarize the projected future status of a company based on the current financial statements and company's forecast. These statements can be done annually to represent forecasting results for the future periods, or whenever the need arises. For example, if a company decides to build a new factory, pro forma statement may be created to reflect the changes to company's profitability, cash flows and financial position as a result of undertaking this project.

Pro forma financial statements are used internally by the business and they are not required to be released to public or to shareholders. They are used to aid managers in making decisions regarding company's operations and financing, as well as to determine areas that need to be analyzed and resolved for the company to continue running smoothly. For example, management may use pro forma financial statements to determine the effect of decreasing the company's inventory to decrease storage costs. The money from the excess inventory will then be invested. On the pro forma balance sheet, the financial position of the company will remain unchanged: the decrease in inventories will be offset by an increase in long-term investments. However, when analyzing the financial ratios, company's management may notice that cutting inventory will result in current assets being too low to finance company's current liabilities. So, the management will have to decide on the course of action to deal with this issue, which would not have been apparent had the pro forma balance sheet not been prepared and analyzed.

#### **Pro Forma Income Statement example**

Absolute Tire Inc. is a company that manufactures tires. For the year 2016, its income statement was:

<b>Absolute Tire Inc.</b>	
<b>Income Statement</b>	
<b>For the Period Ending January 2nd, 2017</b>	
<b>Revenues</b>	
Sales Revenue	\$2,670,000
<b>Expenses</b>	
COGS	\$1,134,750
Selling, general and administrative expense	\$368,000
<b>Income before Taxes</b>	<b>\$1,167,250</b>
<b>Income Tax</b>	<b>\$291,812</b>
<b>Net Income</b>	<b>\$875,438</b>

Company's sales depend on the orders from the auto retailer company Street Cars Ltd. in the city, and sales in the local store. The sales in the local store have been constant for the past 5 years, averaging 2000 tires sold per year. In 2016, the company sold 11350 tires to Street Cars Ltd. as per company's orders. The company sells tires for \$200 each and will keep the same price in 2017. For 2017, Street Cars Ltd. is expecting to host an auto event and the management of the company has increased their order to 18500 tires for 2017. Due to the event, Absolute Tire Inc. expects an increase in demand for their tires sold in the local store to go up by 30%. Production costs per item remain the same. Due to an increase in company's operations, selling, general and administrative expenses are expected to increase by 20%. The company pays 25% in federal and provincial taxes.

What is the pro forma income statement for Absolute Tire Inc. for 2017?

**Solution**

To construct the pro forma income statement, we need to determine sales and expenses for 2017.

We begin by projecting the sales. In 2016, company's sales were \$2,670,000. To generate these sales, Absolute Tire Inc. sold 13,350 tires (2000 in the local store and 11,350 to Street Cars Ltd.), each retailing for \$200. We know that in 2017, company's sales are expected to increase. Street Cars Ltd. has ordered 18,500 tires, which will generate \$3,700,000 in sales. Moreover, the sales in the local store are also expected to increase (by 30%). If on average the company sells 2000 tires per year in the local store, then in 2017 the company can expect to sell 2000 tires \* 130% = 2,600 tires, earning Absolute Tire \$520,000 in sales. So, for 2017, the forecasted sales are:

$$\$3,700,000 + \$520,000 = \mathbf{\$4,220,000}$$

The number of tires sold to generate these sales is 18,500+2,600= 21,100.

In 2016, the company's COGS were \$1,134,750. These are expenses the company incurred to manufacture 13,350

tires. This means, that the production cost per tire was  $\$1,134,750/13,350 \text{ tires} = \$85/\text{tire}$ . The company expects the production costs per tire to be the same in 2017. So, to produce 21,100 tires in 2017, the company will have

$\text{COGS} = 21,100 \text{ tires} * \$85/\text{tire} = \mathbf{\$1,793,500}$  in production expenses.

Selling, general and administrative expense is expected to increase by 20% in 2017 compared to 2016. So, in 2017, these will be:

$\$368,000 * 120\% = \mathbf{\$441,600}$

Now that we have expected revenues for the period and expenses, we can calculate Absolute Tire's income before tax:

$\text{Income before tax} = \text{Sales revenue} - \text{COGS} - \text{Selling, general and administrative expense} = \$4,220,000 - \$1,793,500 - \$441,600 = \mathbf{\$1,984,900}$ .

Since company pays 25% in taxes, the income tax for 2017 is estimated to be:

$\text{Income tax} = \text{Income before taxes} * \text{Income tax rate} = \$1,984,900 * 25\% = \mathbf{\$496,225}$

Now that we have the income before taxes and the income tax amount, we can calculate Absolute Tire's net income as follows:

$\text{Net income} = \text{Income before tax} - \text{Income tax} = \$1,984,900 - \$496,225 = \mathbf{\$1,488,675}$ .

So, the pro forma income statement for 2017 is:

<b>Absolute Tire Inc.</b>	
<b>Pro Forma Income Statement</b>	
<b>For the Period Ending January 2nd, 2018</b>	
<b>Revenues</b>	
Sales Revenue	\$4,220,000
<b>Expenses</b>	
COGS	\$1,793,500
Selling, general and administrative expense	\$441,600
<b>Income before Taxes</b>	<b>\$1,984,900</b>
<b>Income Tax</b>	<b>\$496,225</b>
<b>Net Income</b>	<b>\$1,488,675</b>

So, as we see from the pro forma income statement, the projected net income for 2017 is expected to be higher than the net income in 2016 by 70% ( $\$1,488,675/\$875,438 = 170\%$ ).

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## 2.4 Tax and Depreciation

In this section, we will explore the topics of income tax and depreciation. They both play important roles in financial decision making and reporting, and it is important to understand how the two are calculated and how they are related to a company's performance.

### 2.4.1 Income Tax

*Income tax* is the amount of money remitted to the government, which is usually a percentage of a company's income. This percentage is called the **tax rate** and is set by the government.

In Canada, companies are taxed on the federal and provincial levels. Tax rates depend on company's income.

- On the federal level, there are two tax rates: the general tax rate, 15%, and small business tax rate, which is 10.5%. The small business tax rate is applicable when a company's income is less than \$500,000, which is also referred to as the *business limit*. That is, if a company generated income lower than the \$500,000 income threshold, then the company has to remit 10.5% of its income in taxes. If more than \$500,000 was generated in income for the tax year, then the company will have to remit 15% of its income to the government.
- On the provincial or territorial level, the two tax rates are – a lower rate and a higher rate. The lower rate applies to small business with income less than \$500,000. The higher rate applies to all other businesses. For example, in Saskatoon, small business has to remit 2% of its income in taxes, while businesses that earned more than \$500,000 have to pay 12%.

Note that these tax rates are for 2017 and may be changed by the government for subsequent years.

The tax is calculated on the *income tax return*, which is based on the information from financial statements and is submitted to the government on the yearly basis. In Canada, income taxes are paid by corporations. A corporation is a business owned by one or more owners. The owners are known as shareholders. A shareholder owns shares of the corporation. Shares are units of ownership in a corporation. For example, if a corporation has 1,000 shares, there may be three shareholders who own 700 shares, 200 shares, and 100 shares respectively. The number of shares held by a shareholder represents how much of the corporation they own. The first shareholder who owns 700 shares owns 70% of the corporation ( $700/1,000 = 70\%$ ).



Note that proprietorships and partnerships are not taxed the same way as corporations are. A proprietorship is a business owned by one person. It is not a separate legal entity, which means that the business and the owner are considered to be the same. For example, the profits of a proprietorship are reported on the owner's personal income tax return, so they are taxed as income of the person using tax rates determined for personal income taxes. A partnership is a business owned by two or more individuals. Like the proprietorship, it is not a separate legal entity and the partners need to report their respective profits on their personal income tax return. Unlike the proprietorship and partnership, a corporation is a separate legal entity. This means, for example, that from an income tax perspective, a corporation files its own tax return (Annand, 2016).

In Canada, all resident corporations (except tax-exempt Crown corporations, Hutterite colonies and registered charities) have to file a corporation income tax return every year even if there is no tax payable. This includes: non-profit organizations; tax-exempt corporations; and inactive corporations. A non-resident corporation has to file a return if, at any time in the year, one of the following situations applies: it carried on business in Canada; it had a taxable capital gain; or it disposed of taxable Canadian property.

Tax is calculated on the income reported on the company's income statement, which is recorded as *income before tax*, which we have already mentioned in section 2.1. Recall that to calculate the income before tax, we must subtract production cost, operating and non-operating expenses. These are also called *tax deductions*, as they reduce taxable income. A company may also be eligible for *tax credits*, which reduce a company's tax amount. Notice the difference: tax deductions reduce the income before tax, while tax credits reduce taxes payable. For example, a tax credit of \$1,000 reduced taxes owed by \$1,000, regardless of the tax rate. A tax credit may be granted in recognition of taxes already paid, as a subsidy, or to encourage investment or other behaviors. For example, a city may offer a local business up to \$1 000 tax credit for the installation of energy saving windows on their properties.

Once income tax is calculated based on the income before taxes reported by the company, it is then subtracted from the income before taxes to obtain company's net income for the period.

For more information on corporation's income tax you can refer to the income tax guide published by the Canada Revenue Agency (a Canadian federal agency that administers tax laws in Canada), <https://www.canada.ca/content/dam/cra-arc/migration/cra-arc/E/pub/tg/t4012/t4012-16e.pdf>.

Let's go back to Canadian Tire income statement. Income tax expense is determined by multiplying a businesses income before taxes by its tax rate. We can determine Canadian Tire's tax rate by dividing its income tax expense by its income before taxes to determine a tax rate. So, Canadian Tire has paid  $\$265.4/\$1001.3 = 0.265$  or 26.5% in 2015 fiscal year. That is, the company paid 15% in federal taxes and 11.5% in provincial taxes.

### 2.4.2 Depreciation

As we already know, a company can deduct operating costs from operating revenues before calculating its taxes payable. There is a particular kind of operating cost that needs to be handled carefully, and that is the operating cost associated with equipment lasting longer than a year. For example, in calculating the operating costs for a

photocopy shop, the paper that needs to be purchased every week is clearly an operating cost. But the shop may also need to buy a new photocopier every five years. How is this cost handled?

Here, five years is the **economic life** of a photocopier. Economic life, which is usually expressed as a number of years, is the total time that the asset is used by the business to generate income. It is also the period over which **depreciation** is charged. Depreciation is the reduction in the value of an asset. The potential causes for depreciation of an asset include

- *physical depreciation*, or wearing out;
- *functional depreciation*, as when the function an asset provides becomes inadequate or unneeded;
- *technological depreciation*, where other means of performing the same function better or more cheaply become available;
- *depletion*, as in the consumption of a non-renewable natural resource; and
- *monetary depreciation*, that is, the need to set aside additional money to replace an asset because inflation has pushed up the price of a replacement.

A photocopier might need to be replaced every 5 years because of physical and technological depreciation. Here, these are the primary reasons photocopier as an asset loses its value. In other words, you can think of this as company using up a part of an asset to produce revenues. From previous example, this would be equivalent to saying that a company “uses up” a photocopier throughout 5 years. After the economic life of an asset is over, the photocopier will be “used up” in production, and its book value will be equal to 0.

Now that we know what depreciation is, how do we approach the its calculations? There are three methods of calculating depreciation that will result in extinguishing the book value of the asset when its estimated useful life ends: the straight-line method, the declining balance method, and the activity-based depreciation method.

### **Straight-Line Method**

The straight-line method reduces the book value of an asset by the same amount each period. This amount is determined by dividing the total value of the asset, less its salvage value, by the number of periods in its useful life. This amount is then deducted from income in each applicable period. Straight-line depreciation is the simplest and most-often-used technique. The depreciation amount for the period is calculated as follows:

ADD EQN (2.24)

For example, say a photocopier costs \$6000. The economic life of a photocopier is 5 years. The residual value of a photocopier after 5 years will be \$1000. Then, using formula 2.10, the yearly depreciation expense is:  $(\$6000 - \$1000) / 5 = \$1000$ . This means that every year for 5 years (from the moment of purchase until the photocopier is disposed) the company can record depreciation expense of \$1000 as part of its production costs. So, the depreciation schedule is as follows:

Year	1	2	3	4	5
<b>Depreciation</b>	\$1000	\$1000	\$1000	\$1000	\$1000
<b>Remaining Balance</b>	\$5000	\$4000	\$3000	\$2000	\$1000

*Table 2.1 Straight-Line Depreciation Schedule*

The last row of the table above shows the remaining balance of an asset, which is the remaining value of an asset after deducting depreciation for the period. It is calculated by subtracting depreciation from the value of an asset at the beginning of the period. To get the remaining balance in the first year we subtracted \$6000-\$1000=\$5000. In the second year, the remaining balance is the value of the asset at the beginning of the period minus depreciation \$5000-\$1000 = \$4000. The remaining value of an asset in the straight-line depreciation method is the residual (or salvage) value.

### **Declining Balance Method**

The declining balance method of depreciation provides for a higher depreciation expense in the first year of an asset's life and gradually decreases expenses in subsequent years. This may be a more realistic reflection of the actual expected benefit from the use of the asset because many assets are most useful when they are new. Under this method, the annual depreciation expense is found by multiplying the residual value of the asset each year by a fixed rate. Since this remaining value will differ from year to year, the annual depreciation expense will differ as well, unlike in the straight -line method. Since the declining balance method will never fully amortize the original cost of the asset, the salvage value is not considered in determining the annual depreciation.

Annual depreciation expense using declining balance method is calculated as:

$$\text{Declining Balance Depreciation} = \text{Dereciation factor} \times \text{Remaining balance of asset} \quad (2.25)$$

Let's calculate depreciation schedule for the same photocopier now using the declining balance method with the depreciation rate being 35%.

For the first year, the depreciation is:

$$\$6000 \times 0.35 = \$2100$$

The remaining balance after first year is thus:

$$\$6000 - \$2100 = \$3900$$

Continuing to the second year, the depreciation is:

$$\$3900 \times 0.35 = \$1365$$

And the remaining balance is:

$$\$3900 - \$1365 = \$2535$$

We continue these calculations for 5 years to get the depreciation schedule below:

Year	1	2	3	4	5
<b>Depreciation</b>	\$2100	\$1365	\$887.25	\$552	\$383.51
<b>Remaining Balance</b>	\$3900	\$2535	\$1647.75	\$1095.75	\$712.24

*Table 2.2 Declining Balance Depreciation Schedule*

Here, we see that at the end of 5 years the remaining value of the photocopier is \$712.24. This means, that the company wrote off \$5287.76 in depreciation expense over 5 years with depreciation rate equal to 35%. (To compare, the company wrote off \$5000 using single-line depreciation method).

### Activity Based Depreciation Method

This method is less common compared to the single-line and declining balance depreciation methods in Canada. Activity based depreciation method is not based on time, but on a level of activity. That is, when the asset is acquired, its life is estimated in terms of its activity level. This could be miles driven for a vehicle or a cycle count for a machine. Using this method, the depreciation is calculated by multiplying the activity depreciation rate by the activity level of the asset:

$$\text{Activity Based Depreciation} = \text{Annual activity (uit)} \times \text{Activity Depreciation Rate} \left( \frac{\$}{\text{unit}} \right) \quad (2.26)$$

For example, a company that uses a depreciation rate of \$0.2/km for its vehicles would have a depreciation expense of \$100 after 500 km. If the vehicle drove 100,000 km in one year, the depreciation for the year would be  $100,000 \text{ km} \times \$2/\text{km} = \$20,000$ . The remaining balance of the asset in the period is calculated as usual by subtracting depreciation from the value of an asset at the beginning of the period.

### Effect of Depreciation on Taxes

Depreciation is part of production expense and can, therefore, be deducted together with other production costs when calculating income before taxes. In general, the allowable annual depreciation deduction is fixed by the income tax laws. In Canada asset depreciation follows **Capital Cost Allowance (CCA)** regulations.

CCA is the only allowable depreciation expense in Canada. The CRA provides a classification of different types of assets, and a CCA rate and depreciation method for each class of assets. In filling in the tax return, each class must be accounted for separately, as follows:

1. Start with the undepreciated cost of all assets in that class at the beginning of the year.
2. Subtract the proceeds from the sale of any assets in that class that you've disposed of during the year.

3. Add the total allowable cost of asset additions during the year. (This is only 50% of the cost of the assets you've actually purchased during the year; the other 50% is shifted forwards to next year.)
4. Subtract any government assistance payments or investment tax credits. The resulting figure is the undepreciated capital cost (UCC) to be used in tax calculations.
5. Apply the appropriate CCA rate. (To find the appropriate rate, check with the most recent publications of Revenue Canada, or consult a tax lawyer.)

To determine which class an asset falls in, please refer to the classifications found on Canada Revenue Agency's "Classes of depreciable property" on their website at: <http://www.cra-arc.gc.ca/tx/bsnss/tpcs/slprtnr/rprtng/cptl/dprcbl-eng.html>. Note that CCA rates vary from 4% all the way up to 100%.

Let's look into how the CCA is calculated in more detail. Suppose, a company purchased a \$30,000 work vehicle and wants to calculate the CCA deductions for the vehicle. When an capital asset is purchased during a company's fiscal year it can begin being depreciated. As we already mentioned above, to determine the amount of the CCA deduction you begin with the assets Undepreciated Capital Cost.

Partially for simplicity in accounting, and partially to account for the different times during a fiscal year an asset can be purchased, the Canada Revenue Agency uses the *half-year rule*. This rule states that during the fiscal year the asset is purchased only *half* of the initial cost can be used in the CCA calculation. For example, a \$30 000 work vehicle would have an initial UCC of:

$$UCC_{year1} = \text{Initial Cost} \times 0.5 \quad (2.27)$$

$$UCC_{year1} = \$30,000 \times 0.5 = \$15,000$$

This value is then multiplied by its appropriate CCA rate to determine the deductible depreciation expense for the year. Motor vehicles fall under "Class 10.1" which has a CCA rate of 30%. The CCA deduction for this year is, thus, calculated as:

$$\text{CCA Deduction}_{year1} = \text{CCA Class Rate} \times UCC_{year1} \quad (2.28)$$

$$\text{CCA Deduction}_{year1} = 0.30 \times \$15,000 = \$4,500$$

This \$4,500 would lower the company's taxable income and, ultimately, its total income tax expense.

Year by year, assets will continue to be depreciated and depreciation expenses will continue to be deducted from income tax determined by each years UCC. With the CCA expense calculated, we can determine the UCC for the following year. After the first year, the half year rules ceases and the full UCC (or the total initial cost), minus the CCA deduction from the previous year, is used to determine the CCA deduction.

The company's UCC for year 2, in relation to the work vehicle, for the following year would be calculated as following:

$$UCC_{year2} = (\text{Initial Cost} \times 0.5) + (UCC_{year1} - \text{CCA Deduction}_{year1}) \quad (2.29)$$

$$UCC_{year2} = (\$30,000 \times 0.5) + (\$15,000 - \$4,500)$$

$$UCC_{year2} = \$25,500$$

The company's CCA deduction for year 2 would be calculated as follows:

$$CCA \text{ Deduction}_{year2} = CCA \text{ Rate} \times UCC_{year2}$$

$$CCA \text{ Deduction}_{year2} = 0.30 \times \$34,000 = \$7,650$$

Note that the CCA Deduction for year 2 is larger than year 1. This is because of the affects of the half year rule reducing the amount deductible in the year the asset was purchased. From this point on, the CCA deduction would be reduced due to the UCC continually becoming smaller with each year. The CCA Deduction for a specific year can be generally calculated as:

$$CCA \text{ Deduction}_{year\ x} = CCA \text{ Rate} \times UCC_{yearx-1} \quad (2.30)$$

Businesses usually have more than one asset, and quite often purchase new assets and sell, or dispose, of old assets. As a result, the CCA deduction is determined by the sum of each assets CCA deductions. Furthermore, acquisitions of new assets are added at the half year rule and the disposition of assets reduces the CCA deduction by its selling price. The Canada Revenue Agency provides form *T777: Statement of Employment Expenses* for individuals and *T2125 Statement of Business or Professional Activities* to calculate and report CCA deductions for the year. The following is an excerpt from *T2125 Statement of Business or Professional Activities*:

1	2	3	4	5*	6	7	8	9	10
Class no.	Undepreciated capital cost (UCC) at the start of the year	Cost of Additions in the year (see B and C below)	Proceeds of dispositions in the year (see areas D and E below)	UCC after additions and dispositions (col. 2 plus col. 3 minus col. 4)	Adjustments for current year additions $1/2 \times (\text{col. 3 minus col. 4})$ . If negative enter "0"	Base amount for CCA (col. 5 minus col. 6)	Rate (%)	CCA for the year (col. 7 x col. 8 or an adjusted amount)	UCC at the end of the year (col. 5 minus col. 9)
						Total CCA claim for the year →			

\*Note: this excerpt does not include all the information necessary to record CCA tax expenses; it is the cumulative section which is used to calculate the CCA Deduction and the UCC\*

(T2125 Statement of Business or Professional Activities, 2016)

Let's continue the truck example to illustrate how the company would depreciate the truck it purchased over the first five years:

Year	Undepreciated capital cost (UCC) at the start of the year	Rate (%)	Capital Cost Allowance (CCA) deduction for the year
1	\$ 15,000	0.3	\$ 4,500
2	\$ 25,500	0.3	\$ 7,650
3	\$ 17,850	0.3	\$ 5,355
4	\$ 12,495	0.3	\$ 3,749
5	\$ 8,747	0.3	\$ 2,624

*Table 2.3 Vehicle Depreciation*

Once an asset has meet the end of its useful, or is otherwise sold, the proceeds from the disposition of that asset are then deducted from the total value reported for asset acquisitions for that year. If the company was to sell the truck and buy a new work vehicle, the proceeds from the first truck's sale would be subtracted from the purchase cost of the second truck when determining the total UCC for the year.

#### Supplementary Material

- For more information on Canadian taxation go to CRA webpage <https://www.canada.ca/en/services/taxes/income-tax/corporation-income-tax.html>

#### Discussion and Review Questions

1. What is the purpose of an income statement, balance sheet and cash flow statement and what information do these financial statements contain?
2. What are financial ratios?
3. What are the categories of financial ratios and what does each category analyse?
4. What are pro forma financial statements and how are they used in forecasting?
5. What is income tax and how is it calculated in Canada?
6. What is depreciation and what methods are used to calculate it? How is it recorded in Canada?

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# Chapter 3: Time Value of Money

## 3.0 Introduction

In 1961, a box of Cheerios cost roughly 25¢. Today, a regular-sized box of Cheerios can cost as much as \$5.00 at a grocery store or supermarket. What is the reason for the price increase, when the actual cereal hasn't changed much in over 50 years? Here are some other questions to think about regarding this scenario:

- How much is a regular (non-collectable) 1961 quarter worth today?
- If you kept that quarter in your sock drawer until today, how much money would you have now?
- If you put the quarter in the bank in 1961, how much would be in the account today (assuming no bank fees or other transactions)?
- Alternatively, if you had purchased 25¢ of stock in Bell Canada Inc. in 1961, how much would your investment be worth today?
- Could you buy a box of Cheerios for 25 cents today? Why?

The answers to these questions lie in the concept of the **time value of money (TVM)**: *a dollar today is worth more than a dollar in the future.*

The time value of money is a fundamental concept in engineering economics. It forms the basis for project decision making and is taken into account every time financial decisions are made. The two concepts associated with the time value of are:

- Inflation/deflation
- Interest

These concepts will be discussed in detail in this chapter.



### Key Concepts and Terms

- Inflation and deflation
- Simple vs. compounding interest
- Effect of time on the economic value of money
- Principles of economic equivalence
- Common types of cash flow series

### Learning Objectives

After completing this chapter, students should be able to:

- Explain the concept of the time value of money
- Discuss inflation and interest as well as their applicability to financial decisions
- Explain the difference between simple and compounding interest and know how to calculate them
- Identify common cash-flow series
- Evaluate economic equivalence

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## 3.1 Inflation

Inflation is one reason why a dollar today is worth more than a dollar in the future. The term **inflation** refers to an increase in the level of prices [1] of goods and services in an economy over time. The **inflation rate** is a percent increase in the price level, usually measured annually. For example, a 2.5% inflation rate implies that the level of prices has increased by 2.5% over the year. Conversely, **deflation** is a decrease in the level of prices.

When the price level rises, each unit of currency (e.g. one dollar) buys fewer goods and services. Consequently, inflation reflects a reduction in the purchasing power per unit of money. (Boundless Economics, 2016). **Purchasing power** is a measure of how much can be purchased with a given amount of money. For example, does \$10 buy you 20 liters of gas or only 10? The purchasing power of \$10 can change. It can change for a number of reasons, including inflation. It can also be affected by geographical area or even from store to store.

Inflation (or deflation) can be a result of an increase in demand (***demand-pull inflation***), or a decrease in the supply of goods and services (***cost-push inflation***).

Let's look at ***demand-pull inflation***. This happens when people want more of a product than is actually available. Consequently, this pulls the prices up, causing inflation. In general, demand-pull inflation occurs almost naturally in a "good" economy. To explain further, in a good economy with low unemployment, people have good jobs and job opportunities, and their wages increase over time. When employees' wages increase, they have more money to buy goods and services, so demand goes up. If demand for goods and services increases, then the people who provide them can charge more, resulting in a price level increase over time. At this point, the cycle can feed itself, as price inflation will allow businesses to hire more staff (lowering unemployment) and leads (again) to employees requiring higher wages.[2]

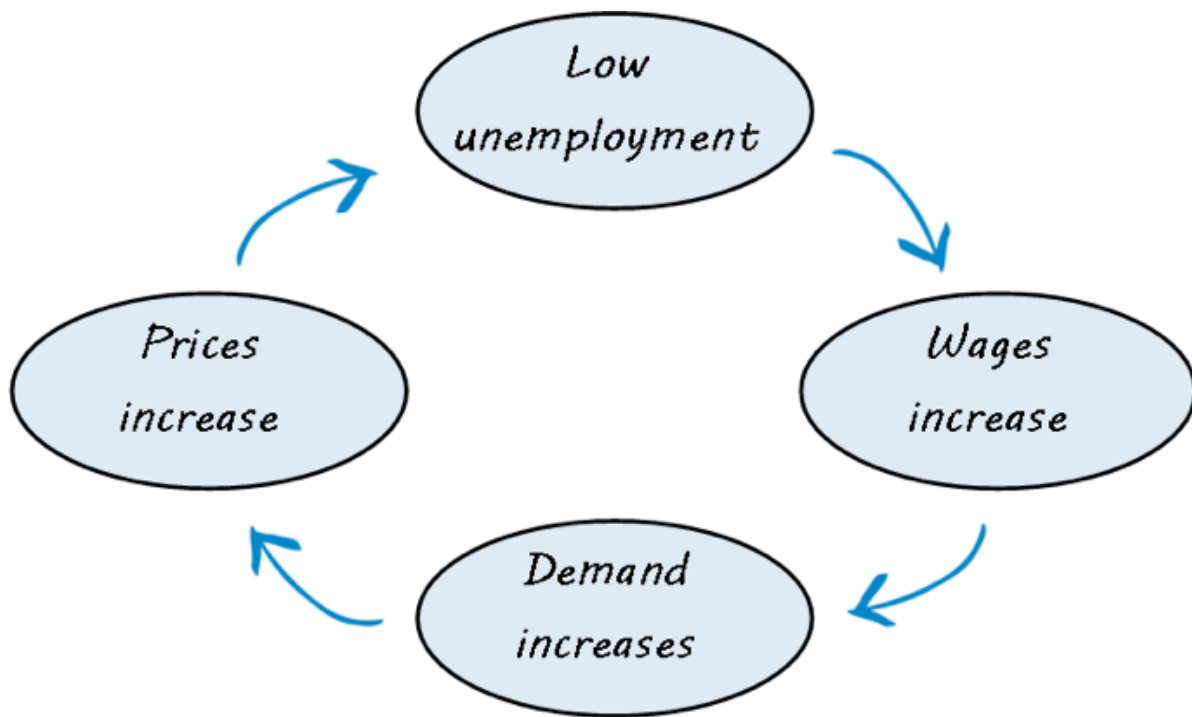


Figure 3.1 The Basic Cycle of Demand-Pull Inflation

Demand-pull inflation can also be a product of monetary policy of a central bank. For example, if the Bank of Canada decides to print more money, demand for goods and services can rise because people have now more money and, thus, are willing to buy more.

Now let's look at **cost-push inflation**. It is caused by a decrease in supply of goods and services, which leads to higher prices of goods and services. Supply decreases can occur due to numerous reasons, e.g. shortage of raw materials, shortage of available labour, crop losses and natural disasters. For example, in 1970s several Middle Eastern oil producing countries (referred to as OPEC) refused to sell oil to Canada, Japan, the Netherlands, the United Kingdom and the United States. As a result, oil prices quadrupled, which made the cost of gas and many other oil products dramatically increase.

Most economies are undergoing modest inflation at all times. However, occasionally an economy may experience extremely high inflation rates, called **hyperinflation**. This is considered to be a disastrous phenomenon. It has only occurred a handful of times in history. Two examples are Germany between 1921 and 1923 and Zimbabwe between 2007 and 2009. In both cases, the federal governments were struggling to repay their debts, so the central banks began printing more money in an attempt to pay off their debts. As we saw earlier, printing money can lead to demand-pull inflation.

Shortly after World War I ended, Germany had to pay their war debt. To do so, the German central bank started printing money (at that time German currency was called Marks). As a result, the German mark- U.S. dollar exchange rate rose from 4.2 to 1 in 1914 to around 4.2 trillion marks to 1 dollar by November 1923. At hyperinflation's peak, prices were rising so fast that waiters had to climb on tables to call out new menu prices in restaurants every half hour. Workers had to bring wheelbarrows with them to work to collect their daily pay.[3]

In November 2008, Zimbabwe had an inflation rate of 79.6 billion percent – equivalent to 98% per day. Every day, prices essentially doubled. Prices were adjusted several times per day. At one point, a loaf of bread cost 550 million Zimbabwean dollars (up from Z\$5.10 five years earlier) (Anonymous, 2006), and teachers were paid in trillions of dollars per month. (Taylor, 2014).



A bill worth 100 billion Zimbabwean dollars. CCBY – Peat Bakke via <https://www.flickr.com/photos/misttermoss/2899197879> accessed Aug.24, 2017

To sustain economic stability and avoid such extreme cases, central banks generally try to keep inflation at a low and stable rate. For example, the Bank of Canada target inflation rate is generally 2%[4].

### 3.1.1 Measuring Inflation

The most common method of measuring inflation is by using the concept of a **basket of goods** – a relatively fixed set of goods and services. The cost of that basket of goods is estimated and compared from time to time. The combination of goods and services in the basket of goods are selected to represent a typical household's expenses. Food and housing, for example, constitute a large share in average households' expenses, thus, changes in food and housing prices affect people's standard of living, or total costs, more than changes in the price of, say, luxury cars (Statistics Canada, 2015). Based on the basket of goods the **Consumer Price Index (CPI)** is calculated (Taylor, 2013). The CPI represents the level of prices in the country at a certain period of time and is used to calculate the inflation rate (or deflation rate). [5]

For example, if we want to calculate the inflation rate between 1961 and 2016, we divide the difference in the CPI values between the two years by the CPI value in 1961:

$$\text{InflationRate} = \frac{CPI_{2016} - CPI_{1961}}{CPI_{1961}} 100\% = \frac{128.4 - 15.7}{15.7} 100\% = 718\% \quad (3.1)$$

In this formula, 1961 is called the *base year* and 2016 is the *target year*.

Let's tie this concept back to our Cheerios example. As mentioned at the beginning of the chapter, a box of Cheerios cost 25¢ in 1961, and a comparable box of Cheerios cost around \$5.00 in 2016. This represents a price increase of 1900% over the 55-year period. The inflation rate of 718% affected this price increase, but shows that the price of Cheerios rose more than the average price of goods over this period.

### 3.1.2 Nominal vs. Real Value/Purchasing Power

Say you want to buy your morning coffee, and the coffee costs \$2.00. You look in your wallet and find two toonies – one minted in 1996, and one minted in 2015. Does it matter which one you use to pay for your coffee? No. Both coins are worth exactly \$2.00. We say the coins have the same **nominal value** – in this case, the nominal value is \$2.00. The nominal value is the numerical value printed on the money, and it *never changes*.

What *does* change about the toonie is its **real value** or **purchasing power**. A toonie will be less useful to you ten years from now than it is today. If you leave your toonie on your nightstand for ten years, you might find that you can no longer buy your coffee with just the toonie. The toonie is still worth \$2.00 (its *nominal value* hasn't changed), but its *purchasing power* has decreased, because it can no longer buy as much as it could ten years ago.

One way to calculate the real value/purchasing power of money is by using a variation of the inflation formula introduced above. For example, to find the purchasing power of \$100 from 1961 (base year) in 2016 (target year), we multiply the CPI ratio of the target year to the base year by the dollar amount in the base year:

$$\frac{CPI_{2016}}{CPI_{1961}} * Dollar Amount_{1961} = \frac{128.4}{15.7} * \$100 = \$817.83 \quad (3.2)$$

As we see, the purchasing power of \$100 from 1961 is equivalent to the purchasing power of \$817.83 in 2016. So, if you spent \$100 in 1961 you would be able to buy a lot more than if you spent \$100 in 2016. In fact, you would be able to buy an equivalent of \$817.83 in 2016 dollars.

For example, in 1960 you could have bought a sofa with \$189.95 from Sears. In 2017, a similar sofa sells for \$1400-\$1700 at Sears (almost 10 times as much!). As we can see, inflation led to a significant increase in prices over the years.

Thus, since the real value of money changes over time due to inflation, we say that a *dollar today is worth more than a dollar tomorrow*. This illustrates the concept of the “**time value of money**”.

It is important to note that inflation affects money management and investing decisions. If the prices of your daily needs – such as food and transportation – increase, you will need a higher income to maintain the same standard of living. Keeping money “under the mattress” does not protect its purchasing power, as inflation will gradually decrease the amount of goods and services you can buy with that money. In contrast, if money was put into an account with a 2% interest rate (that is, the initial deposit will increase by 2% per year), and the inflation rate was also 2%, the account's purchasing power would remain the same after one year. The balance in the account would

have increased to maintain the purchasing power. So, as we see, **applying interest can help mitigate the effects of inflation**, making interest an important component of the TVM concept.

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[1] *When discussing inflation and deflation, we use the term “level of prices,” which represents the general prices of goods and services. To assess inflation, we need to measure a change in the level of prices, not just price fluctuations of individual goods, because these prices can fluctuate due to a company’s policy, marketing strategy and other short-term effects. Hence, these fluctuations do not necessarily represent inflation. Inflation (or deflation) is intended to represent the longer term trends in prices.*

[2] (From Khan Academy. Not sure about their TOU.)

[3] <http://www.economist.com/blogs/freeexchange/2013/11/economic-history-1>

[4] <http://www.bankofcanada.ca/rates/indicators/key-variables/inflation-control-target/>

[5] For more information about inflation and CPI in Canada, visit [inflationcalculator.ca](http://inflationcalculator.ca) or [statcan.gc.ca](http://statcan.gc.ca).

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## 3.2 Interest

Two services typically offered by banks are savings accounts and loans. If you deposit money into a savings account, the amount of money in the account gradually increases. At a later date, you can withdraw more money than you deposited. In contrast, when you borrow money in the form of a loan, the total amount you will have to repay will be more than the amount you originally borrowed. In both of these scenarios, an increase in the amount of money is due to interest.

**Interest** is the cost of borrowing money. It is the amount of money that is paid in addition to the amount borrowed, loaned or invested. Just like you pay to rent a car or rent an apartment, you pay to “rent” money. The original amount borrowed, loaned or invested is called the **principal**. Interest is charged on the principal due to the following factors:

- **Inflation.** As noted in the section 3.1, due to inflation money’s purchasing power slowly decreases. Without charging interest, the lender would be left with less purchasing power than they started with.
- **Risk.** There is a chance that the borrower will default on (not pay back) their loan. To account for this risk, lenders charge interest. The interest they earn helps cover losses from borrowers who are unable to make their payments.

An **interest rate** is a percentage rate applied on the principal, which is used to calculate the amount of interest generated in an **interest period** (the frequency that interest is calculated, e.g. monthly, bi-weekly, yearly).

There are two types of interest: **simple interest** and **compound interest**.

### 3.2.1 Simple Interest

**Simple interest** is calculated on the amount that was originally borrowed, loaned or invested – the principal. Interest accumulated in previous periods does not earn additional interest.

### Example 3.1

Suppose you make a deposit of \$100 in a bank account that pays 5% interest per year. After one year, you earn 5% interest, or \$5, bringing your total balance to \$105. After one more year, since simple interest is paid only on your principal, you again earn 5% of the original \$100. That means you earn another \$5 in the second year, and will earn \$5 for every year of the investment. (Boundless Finance, 2016). The diagram below shows the interest the \$100 deposit earns each year and how that affects the total value of your deposit. The next table shows account balances for this scenario for the first 5 years.

Year	Beginning Balance	Interest Earned	Ending Balance
0	—	—	\$100.00
1	\$100.00	\$5.00	\$105.00
2	\$105.00	\$5.00	\$110.00
3	\$110.00	\$5.00	\$115.00
4	\$115.00	\$5.00	\$120.00
5	\$120.00	\$5.00	\$125.00

Table 3.1 Simple Interest Earned on a \$100 Deposit

Typically, the current account balance is called the **present value** (\$100 in period 0) and the account balance at some point in the future is termed the **future value** (\$125 in period 5).

The future value consists of the present value (principal) and the total interest:

$$F = P + I \quad (3.3)$$

Where  $F$  = Future Value

$P$  = Present Value

$I$  = Total Interest

Total interest  $I$  is the **total simple interest** paid on the principal in all periods. It is calculated as follows:

$$I = (iP)N \quad (3.4)$$

Where  $P$  = Present value

$i$  = interest rate

$N$  = number of periods



Thus, the future value formula becomes:

$$F = P + (iP)N$$

Rearranging, we get the **future value with simple interest formula**:

$$F = P(1 + iN) \quad (3.5)$$

Applying this formula to example 3.1 we get:

$$F = \$100(1 + 0.05 * 5) = \$125$$

So, the future value of the investment in 5 years is \$125. Total interest paid is \$25.

While simple interest is a useful theoretical concept, it is rarely used in the real world. Loans you get from a bank, money you invest in savings accounts, credit card debt etc. accrue what is called *compound interest*.

### 3.2.2 Compound Interest

**Compound interest** is calculated on the total amount – the principal and previously earned interest – in a given period. In other words, compound interest includes *interest earned on interest*, not just interest earned on the principal, like simple interest does. The interest period for compound interest is typically termed the **compounding period**. For the same interest rate, compound interest will always result in more total interest than simple interest.

#### Example 3.2

Suppose you make the same \$100 deposit into a bank account that pays 5% interest, but this time, the interest is compounded annually. After the first year, you will again have \$105. At the end of the second year you also earn 5% interest, but this time it is calculated based on your \$105 balance. Thus, you earn \$5.25 in interest in the second year, bringing your balance to \$110.25. In the third year, you earn 5% interest on your \$110.25 balance, or \$5.51. Table 3.2 shows how interest accumulates in the account for the first 5 years.

Year	Beginning Balance	Interest Earned	Ending Balance
0	–	–	–
1	\$100.00	\$5.00	\$105.00
2	\$105.00	\$5.25	\$110.25
3	\$110.25	\$5.51	\$115.76
4	\$115.76	\$5.79	\$121.55
5	\$121.55	\$6.08	\$127.63

*Table 3.2 Compound Interest Earned on a \$100 Deposit*

While you may use a table to calculate the future value, it may be convenient to develop a formula for this calculation.

In year 1, interest is applied to the principal (present value in period 0):

$$I_1 = Pi$$

Thus, the future value in year 1 is:

$$F_1 = P + I_1 = P + Pi + P(1 + i)$$

In year 2, interest is now calculated on the ending balance in year 1, which is  $F_1$ :

$$I_2 = F_1i = P(1 + i)i$$

The Future value in year 2 is therefore:

$$F_2 = F_1 + I_2 = P(1 + i) + P(1 + i)i = P(1 + i)(1 + i) = P(1 + i)^2$$

Similarly, in year 3 interest is calculated on the ending balance in year 2 –  $F_2$ :

$$I_3 = F_2i = P(1 + i)^2i$$

So, the future value in year 3 is:

$$F_3 = F_2 + I_3 = P(1 + i)^2 + P(1 + i)^2i = P(1 + i)^2(1 + i) = P(1 + i)^3$$

Continuing for N periods, we get the general **future value with compound interest formula for single cash flows**:

$$F = P(1 + i)^N \quad (3.6)$$

Remembering that , the formula for **total compound interest** is:

$$I = P[(1 + i)^N - 1] \quad (3.7)$$

To summarize, the formulas for simple and compound interest are in Table 3.3 below.

	Simple Interest	Compound Interest
<b>Total Interest</b>	$I = (iP)N$	$I = P[(1 + i)^N - 1]$
<b>Future Value</b>	$F = P(1 + iN)$	$F = P(1 + i)^N$
<b>Interest earned</b>	Only on principal	Both principal and accumulated interest

Table 3.3 Simple and Compound Interest

Let's see an example problem comparing simple interest to compound interest.

#### Simple and Compound Interest Examples

*Suppose we want to make one deposit into a bank account today in order to withdraw \$2000 five years from now. How much would we have to deposit*

- If the account paid 5% **simple** interest annually?*
- If the account paid 5% **compound** interest, compounded annually?*

#### **Solution**

##### a. Simple Interest

From the problem statement, we know that the future value of the account should be \$2000, so  $F = \$2000$ . We are given the interest rate,  $i = 5\% = 0.05$ , and the number of years,  $N = 5$ . Given all this information, we first use the simple interest formula 3.5 to solve for  $P$  – the value of the initial required deposit:

$$F = P(1 + iN) \rightarrow P = \frac{F}{1+iN} = \frac{\$2000}{1+(0.05)(5)} = \$1600.00$$

#### b. Compound Interest

For the compound interest account, we use compound interest formula 3.6 to solve for  $P$ :

$$F = P(1 + i)^N \rightarrow P = \frac{F}{(1+i)^N} = \frac{\$2000}{(1+0.05)^5} = \$1567.05$$

Thus, comparing the two values, we can see that you would have to deposit a smaller amount into a bank account with compound interest. This is due to the fact that compound interest yields higher total interest amounts compared to simple interest, as noted before. By putting money in a compounding account rather than a simple-interest account, even at the same interest rate, you would earn an extra \$33. Even though \$33 does not seem like much money, it can be significant when considering larger amounts invested over longer time periods. If you were to withdraw \$2 million after five years under the same terms, the difference would be \$33 000!

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## 3.3 Economic Equivalence

We have just considered two key factors relating to the time value of money:

- Inflation, which explains the reduction in purchasing power over time; and
- Interest, which is the cost of borrowing money and helps maintain the lender's purchasing power.

The TVM concept enables us to compare situations when financial transactions occur in different time periods. For example, would you prefer to get \$500 now, \$700 three years from now or \$200 a year for the next three years? How would you decide? Are these three options equal?

When two sets of financial transactions yield the same outcome regardless of the timing of the transactions, the two are said to be **economically equivalent**. In other words, if there were two cash flows that occurred at different times and had the exact same effect on a person's financial position, then the two cash flows would be economically equivalent. In these situations, we are indifferent to choosing one rather than the other.

### Example 3.3

Let's say that Adam can choose to receive \$100 from Becky right now, or \$105 a year from now, and he has access to a bank account that pays 5% interest compounded annually. He would be indifferent when choosing which amount to take: he could take the \$105 a year from now, or he could take the \$100 and deposit it into the bank account to earn \$5 in interest and end up with a total of \$105 in the bank after one year. Thus, the \$100 cash flow and the \$105 cash flow are economically equivalent.

### 3.3.1 Principles of Economic Equivalence

There are **four key principles** that always hold true and are fundamental to understanding economic equivalence and its application to financial decision-making.

**Principle 1: Cash flow comparisons require a common time basis.** Alternative cash flow scenarios must be compared within the same time period. This means we need to “move” cash flows to put them all in the same time period. Often, we compare the alternatives at the present time, but a future point in time may be used as well.

In example 3.3, we compared the two cash flows in the same time period as per principle 1. We applied interest to \$100 in the current period to obtain the future value after one year using the future value formula introduced in the Interest section. This yields the future value of \$105. As principle 1 states, we had to find the future value of the present cash flow for it to be comparable to the other cash flow that occurs one year from now. We then compared the two cash flows, arriving to conclusion that the two are economically equivalent.

**Principle 2: Economic equivalence is dependent on the “discount rate”.** To account for the time value of money while “moving” these cash flows we must *reduce* the economic value of future cash flows. Why? Recall that a dollar today is worth more than a dollar tomorrow, which means that a dollar tomorrow is worth *less* than a dollar today. So, when converting future cash flows to their equivalent (lower) present value to compare these cash flows we use what is termed a **discount rate**. (Note that the term discount rate is also used when converting cash flows to a future point in time.)

**Note:** A “discount rate” is the rate used to discount future cash flows to adjust for the time value of money. Generally, the discount rate is set to account for a cost (or group of costs) the investor or creditor may incur due to the investment. Depending on the context, the discount rate can be equivalent to the interest rate, for example when dealing with savings accounts or loans. Usually, however, the discount rate is higher than the interest rate, as it accounts for other costs as well, such as opportunity costs which will be covered in Chapter 5.

We will talk more about the discount rate and how it is set in chapter 5.

Changing the discount rate that is used to “move” cash flows to other time periods will change values of the respective cash flows. From example 3.3, if the interest rate was 10% instead of 5%, the future value of the \$100 cash flow after one year would be \$110. Comparing with the alternative \$105 cash flow in the same time period, we see, that the two cash flows are no longer economically equivalent. Note, that we use the interest rate as the discount rate in this case to account for the time value of money.

**Principle 3: Comparisons may require conversion to a single cash flow.** When there are multiple cash flows to be considered in an alternative cash flow scenario, the individual cash flows may need to be combined into a single cash flow for comparison. To illustrate this, let’s slightly modify example 3.3.

Say Adam now has to decide whether to receive \$100 from Becky now or, as an alternative, \$60 in the first year and \$45 in the second year, as shown in the diagram below. Like before, he has access to a deposit account that pays 5% compounded annually.

This time, we must compare the single \$100 cash flow to a multiple cash flow alternative. The future value of the \$100 cash flow in 2 years is \$110.25. As per principle 3, the future value of the alternative option is:  $\$60 \times 1.05 + \$45 = \$108$ . So, by converting the multiple cash flows to a single one we can see the two alternatives are not economically equivalent.

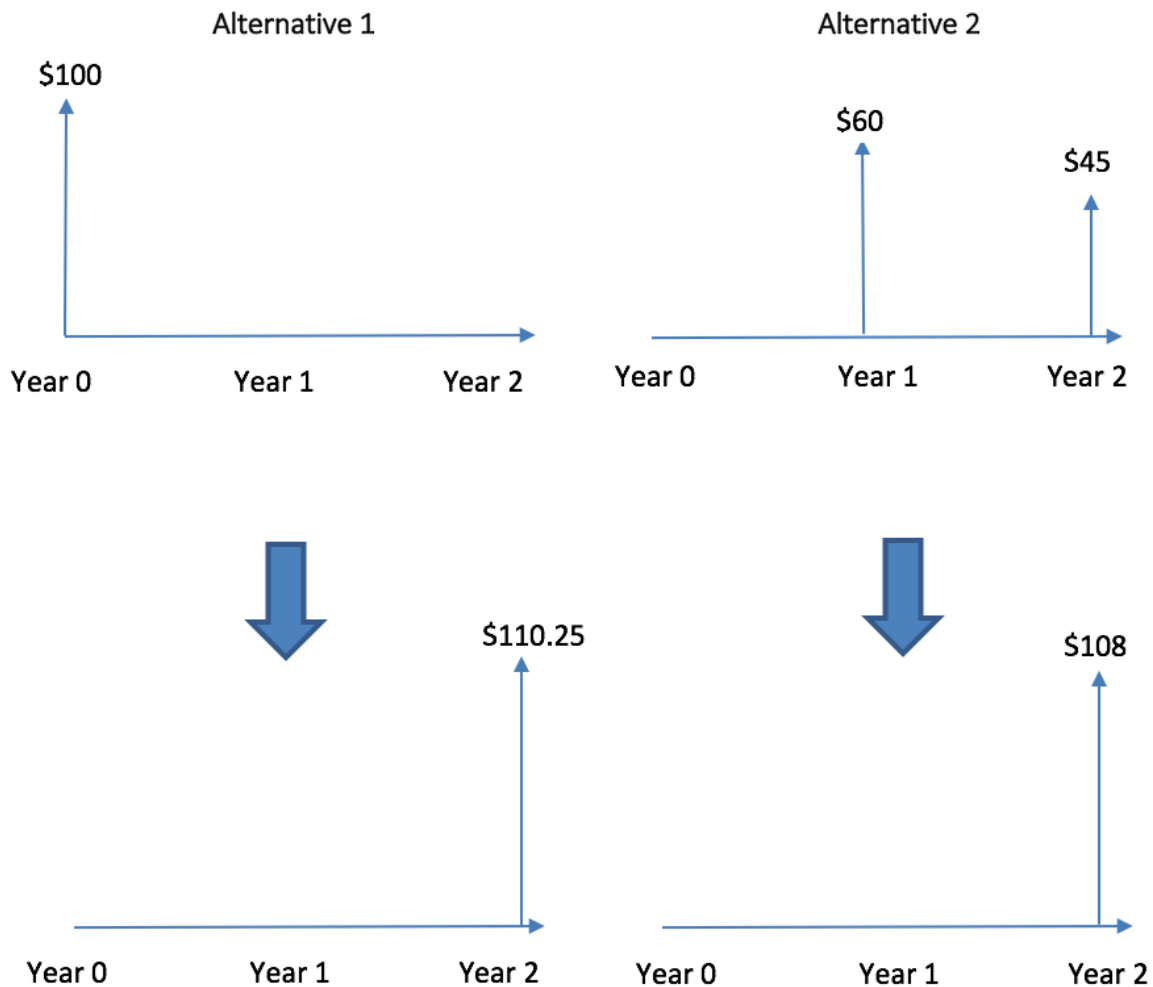


Fig. 3.2 Cash Flow Diagrams for Alternatives

**Principle 4: Equivalence is maintained regardless of point of view.** As long as the same discount rate is used (see principle 2), the equivalence between cash flows will be maintained at any point in time to any party involved in the transactions. That is, it doesn't matter if you are paying or receiving money. Economic equivalence will be the same from both perspectives.

Let's analyze the cash flow diagrams from Adam's and Becky's point of view. From example 3.3 comparing \$100 today and \$105 a year from now with 5% interest compounded annually, as per principle four, the economic equivalence will be maintained for both Adam and Becky.

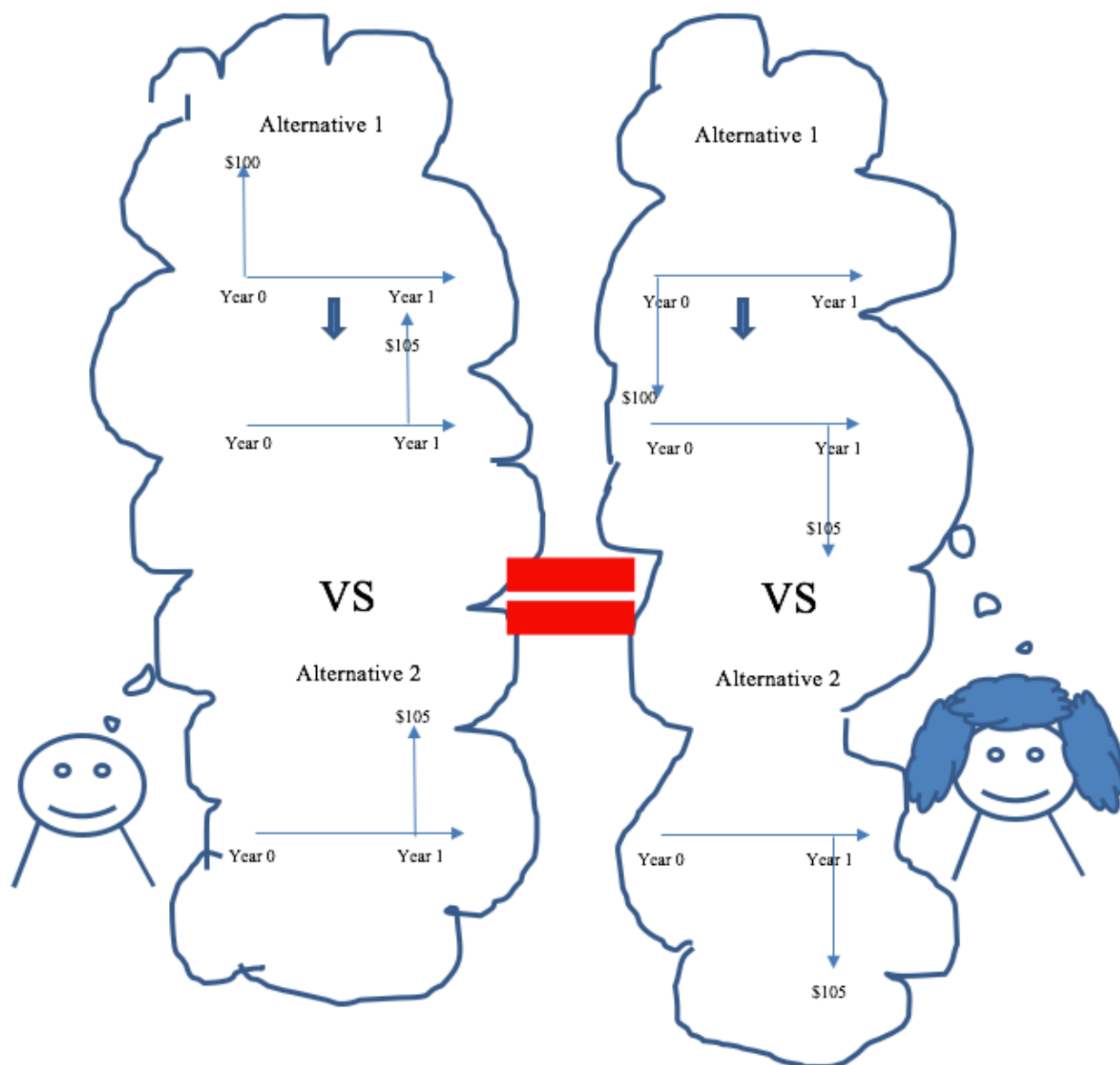


Fig. 3.4 Cash Flow Equivalence for Adam and Becky



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## 3.4 Equations of Economic Equivalence

In section 3.3, we discussed the four key principles of economic equivalence. We need these when analyzing cash flows and evaluating economic equivalence. There are several cash flow patterns that frequently occur. Fortunately, equations have been developed to facilitate the cash flow analysis.

We refer to cash flow patterns as **series**. There are four basic types of cash flow series:

- Uniform series
- Linear gradient series
- Geometric gradient series
- Complex (random) cash flows

In this section, we will take a closer look at each one of these series and their analysis. All of the equations used to analyze each of these types of series are based on the single cash flow equation developed in section 3.2.2 based on the concept of compound interest.

### 3.4.1 Single Cash Flow

Single cash flows involve a single financial transaction at a point in time. An example of a single cash flow is a purchase of a car with a single payment. The alternatives in example 3.3 both represent single cash flows.

The future value formula 3.6 from section 3.2.2 is used to analyze single cash flows:

$$F = P(1 + i)^N$$

Previously in this chapter, this formula was used to calculate the future value of investments, deposits or loans when accruing compound interest. The formula calculates a cash flow's economic equivalent for a given discount rate and a point in time. In this case, we use a discount rate instead of an interest rate, although depending on the context the interest rate may be equal to the discount rate.

The future value formula enables us to “move” cash flows to a future point in time. By rearranging the future value formula we obtain **present value (P) formula**, which enables us to “move” cash flows to the present.

The formula for discounting single cash flows, thus, becomes:

$$P = \frac{F}{(1+i)^N} \quad (3.8)$$

Now, let's look at an example of how single cash flow analysis is performed.

#### Single Cash Flow Example

*You are an employee at Dunder Mifflin Paper Company. Your boss, Michael, offers you two options for a salary bonus: you can either accept a \$1000 bonus now, or you can wait and take a \$1200 bonus two years from now. Which one should you choose if your discount rate is 5% per year?*

##### Solution

We tackle this problem by finding the economic equivalent of the \$1200 bonus – in other words, we use discounting to “move” the \$1200 bonus to current year to find its present value. We then compare the \$1200 bonus to the \$1000 bonus to see which one would be the better choice.

Step 1: Discount the \$1200 bonus for two interest periods (two years) at 5% annually to obtain its present value. Here,  $N = 2$  and  $F = \$1200$ . Thus, using formula 3.8:

$$P = \frac{F}{(1+i)^n} = \frac{\$1200}{(1+0.05)^2} = \$1088.44$$

Step 2: Compare the options. **At a 5% discount rate, the \$1200 bonus has a present value greater than \$1000, so the \$1200 bonus should be chosen.**

### 3.4.2 Uniform Series

A **uniform series**, sometimes called an **equal-payment series**, is a cash flow series in which the same amount of money is paid or received in two or more sequential periods, as shown in Figure 3.4. These types of equal payments are also referred to as **annuities**.

This type of cash flow series is probably familiar to you: many long-term loans, such as house mortgages and car loans, involve a fixed monthly payment for a set length of time. These arrangements allow buyers to spread out payments on large purchases (like cars and houses) that would be difficult for most people to buy in one lump-sum.

An example of a uniform series is illustrated by a cash flow diagram below:

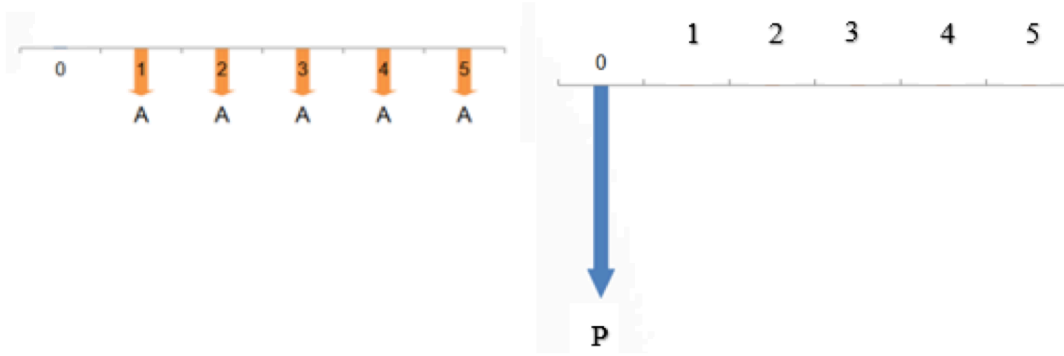


Fig. 3.4 Uniform Cash Flow Series with Negative Cash Flows

We can analyze equal payment series scenarios in four different ways. These formulas can be used to convert uniform cash flows into an equivalent single cash flow, or distribute a single cash flow into an equivalent uniform series. For further information on how these formulas were developed, refer to Section 3.4.7 for the full derivation.

- **Present-value-from-payment formula:** used to calculate the present value,  $P$ , from the regular payment  $A$  (given  $A$ , find  $P$ ).

$$P = A \left[ \frac{(1+i)^N - 1}{i(1+i)^N} \right] \quad (3.9)$$

This formula can be used to answer questions such as “*how much can I borrow to buy a car if I can afford monthly payments of \$500?*”

- **Payment-from-present-value formula:** used to calculate the regular payment,  $A$ , from the present value  $P$  (given  $P$ , find  $A$ ). This is used when distributing a large lump-sum value into smaller, equal payments.

$$A = P \left[ \frac{i(1+i)^N}{(1+i)^N - 1} \right] \quad (3.10)$$

For example, “*how much will my monthly car payment be if I borrow \$10 000 to buy a car now and plan to pay it back over 5 years?*”

- **Future-value-from-payment formula:** used to calculate the future value,  $F$ , from the regular payment  $A$  (given  $A$ , find  $F$ ). It is commonly used to determine how much an account or loan will be worth after  $N$  periods of regular contributions.

$$F = A \left[ \frac{(1+i)^N - 1}{i} \right] \quad (3.11)$$

As an example: “*if I save \$500 every month, how much money will I have 5 years from now?*”

- **Payment-from-future-value formula:** used to calculate the regular payment,  $A$ , from the future value  $F$  (given  $F$ , find  $A$ ). This formula is useful when trying to calculate the size of payment required to have a certain amount,  $F$ , in the future.

$$A = F \left[ \frac{i}{(1+i)^N - 1} \right] \quad (3.12)$$

For example, “if my goal is to have \$10,000 in 5 years, how much will I have to put into my savings account each month?”

**An important note** regarding timeframes: in the above formulas,  $N$  denotes the number of periods in the uniform series,  $i$  denotes the interest (or discount rate) per period, and  $A$  denotes the equal cash flow per period.  **$N$ ,  $i$ , and  $A$  must all be in the same time frame.** If an example requires calculating a monthly payment ( $A$ ), then  $N$  must be months and  $i$  must be the monthly interest rate.

Note that calculating the present value of a uniform series will place the present value one period before payments begin. As shown in the cash flow diagram (Figure 3.4), if payments start in period 1, then  $P$  occurs in period 0.

Let’s look at some examples.

#### Uniform Series Example #1

*Aladdin wants to borrow \$12,000 and pay it off in equal monthly payments over a 5-year period. If the monthly interest rate is 0.75%, compounded monthly, how much would each payment be?*

##### Solution

$N = 5$  years or 60 months,  
 $i = 0.75\% = 0.0075$ , and  
 $P = \$12,000$ .

To solve for  $A$ :

$$A = P \left[ \frac{i(1+i)^N}{(1+i)^N - 1} \right] = \$12000 \left[ \frac{0.0075(1+0.0075)^{60}}{(1+0.0075)^{60} - 1} \right] = \$249.10$$

**Thus, the monthly payment would be \$249.10.** Notice, this means Aladdin ends up actually paying a total of \$14 946 for his \$12 000 loan ( $\$249.10 \times 60$ ), where \$2 746 is the total interest paid to the lender on top of the loan.

#### Uniform Series Example #2

*Brook wants to start saving for retirement. She plans to deposit \$500 per month into a savings account that earns 6% per year (compounded annually). How much money will Brook have in the account after 25 years?*

##### Solution

Here, we have monthly payments and we need to find the future value of the account, so we will use the future-value-from-payment formula 3.11.

But note the units: we are dealing with monthly payments, but interest is applied annually. This means

we must use the **end of period convention** – treat all monthly deposits within each year as if they were all deposited on the last day of the year (discussed in Front material).

So, we have:

$$A = (\$500/\text{month})(12 \text{ months/year}) = \$6000/\text{year}$$

$$i = 6\% \text{ per year}$$

$$N = 25 \text{ years}$$

$$F = A \left[ \frac{(1+i)^N - 1}{i} \right] = \$6000 \left[ \frac{(1+0.06)^{25} - 1}{0.06} \right] = \$329187.07$$

So, after 25 years, Brook will have \$329 187.07 in her account. Note that without any interest she would have only saved up \$150 000 (\$6000 per year x 25 years). Interest more than doubled the money!

### Deferred Annuity Example

Given the schedule of cash flows in Figure 3.5, calculate the present value, **at period 0**, using an annual discount rate of 7%.

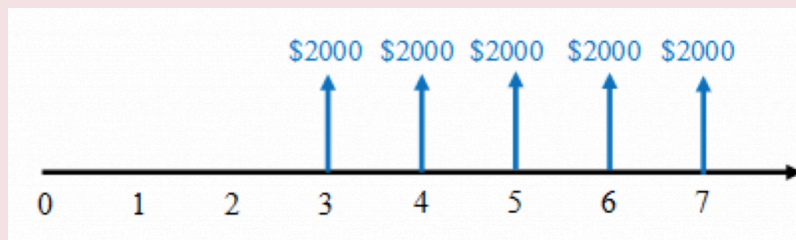


Fig. 3.5 Deferred Annuity Cash Flow Series

#### Solution

In this case, we have  $A$ , and need to find  $P$ , so we will use the present-value-from-payment formula 3.9. But note: this formula will give us the present value in period 2 (see Figure 3.6). We need to then “move” it to period 0 (see Figure 3.7).

Step 1: Find the present value ( $P_2$ ) of the uniform series.

$$A = \$2000$$

$$i = 7\%$$

$$N = 5 \text{ years}$$

$$P_2 = A \left[ \frac{(1+i)^N - 1}{i(1+i)^N} \right] + \$2000 \left[ \frac{(1.07)^5 - 1}{0.07(1.07)^5} \right] + \$8200.39$$

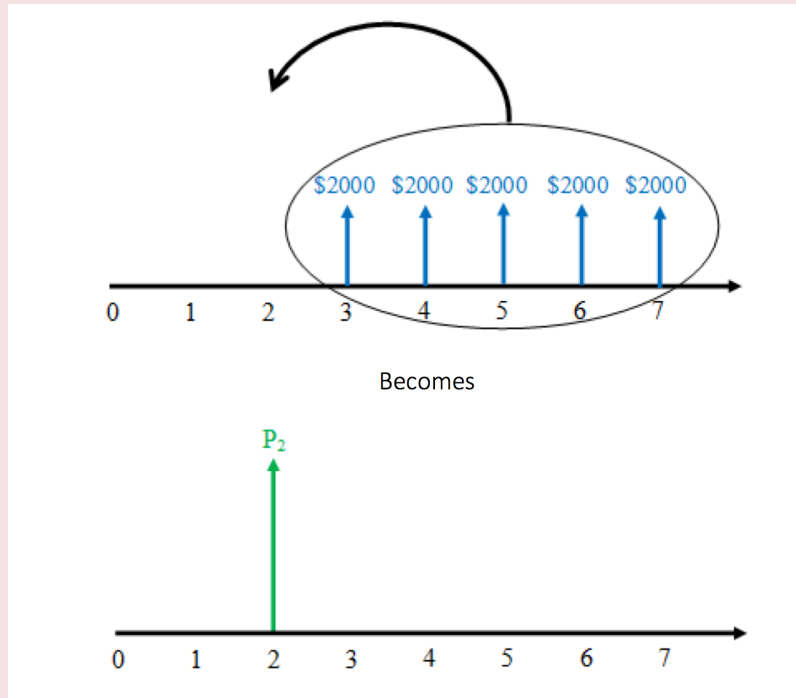
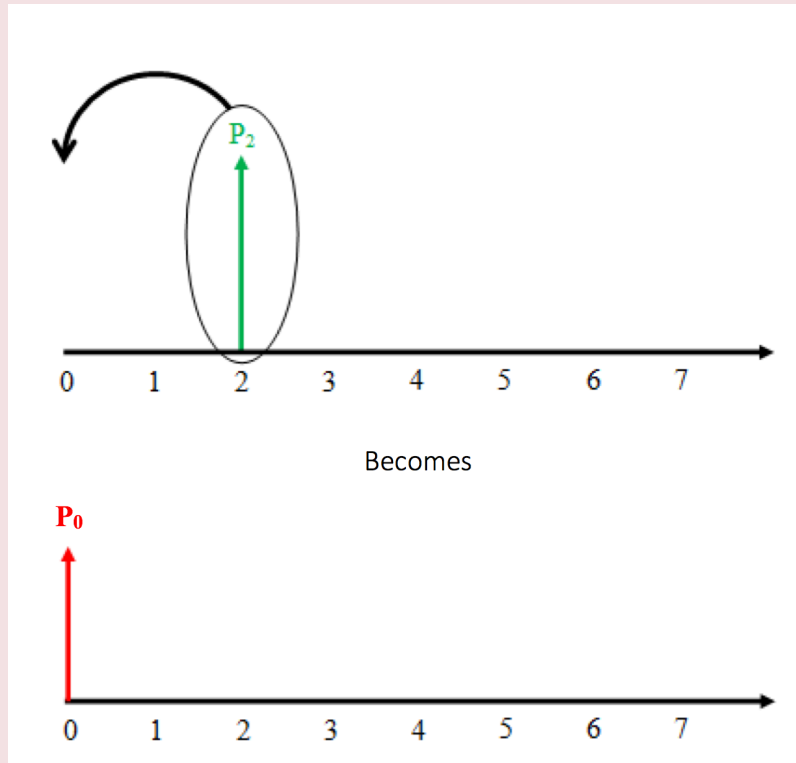


Fig. 3.6 Finding the P2 Cash Flow of the Uniform Series

Step 2: Discount the P2 cash flow to period 0.

Now that we have a single cash flow at period 2, we can discount it to obtain the present value at period 0:

$$P_0 = \frac{P_2}{(1+i)^N} = \frac{\$8200.39}{(1.07)^2} = \$7162.54$$

Fig. 3.7 Discounting of the  $p_2$  Cash Flow

Therefore, the value of the uniform series at period 0 is \$7162.54.

### 3.4.3 Linear Gradient Series

A **linear gradient series** is a series of cash flows which increase or decrease by a constant amount every period. An illustrative example is found in Figure 3.8 below.

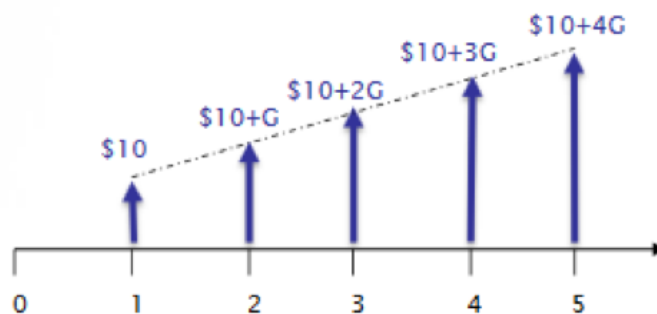


Fig. 3.8 Linear Gradient Series

As we see, in each period the amount is increasing by \$5. This is the **gradient  $G$** . In the cash flow diagram above,

the cash flow in period 1 is \$10. In the second period, we add  $G$  to obtain a value of \$15. In the third period, we add two times the gradient to get a cash flow of \$20... and so on. Note that in Figure 3.8 cash flows are increasing due to a positive  $G$ . However,  $G$  can also be negative, so that each subsequent payment will be smaller than the previous one by  $G$ .

When calculating the linear gradient series, we must **break up the cash flows into two series**: a uniform series and a linear gradient series with a cash flow of zero in period 1, as shown below. In the Figure 3.8,  $A$  would be \$10 and  $G$  would be \$5.

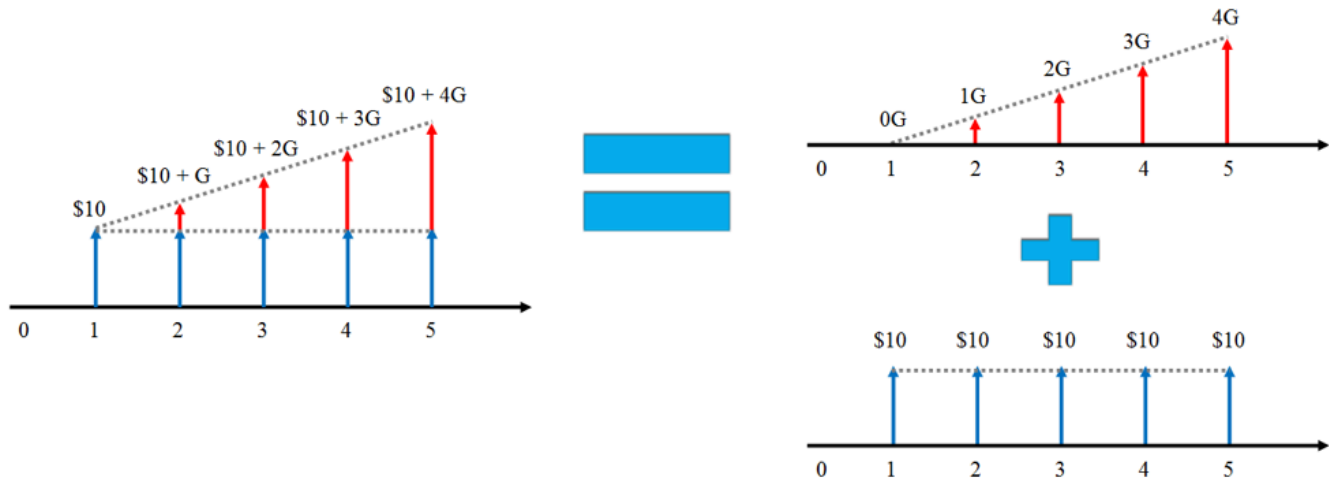


Fig. 3.9 Two-Step Analysis of the Linear Cash Flow Series

Notes on using the linear gradient series:

- As with the uniform series that calculating the present value of a linear gradient series places the present value one period before the cash flows begin; if cash flows begin in period 1, the  $P$  will occur in period 0.
- $N$  is the number of interest periods, including the period in which the gradient's cash flow is zero.
- $G$  can be positive or negative. If the cash flows are increasing each period, then  $G$  is positive; if they are decreasing, then  $G$  is negative.
- $A$  is **always** equal to the cash flow in the first period.

Formulas for analyzing a linear gradient series:

- **Present value of a linear gradient series:** used to calculate the present value (in period 0) of a linear gradient portion of the series beginning in period 1 (given  $G$ , find  $P$ ).

$$P = G \left[ \frac{(1+i)^N - iN - 1}{i^2(1+i)^N} \right] \quad (3.13)$$

This formula along with the corresponding uniform series formula can be used to answer questions such as:



*“what is the present value of a 10 year maintenance contract for which the payments increase by \$1000 per year?”*

- **Future value of a linear gradient series:** used to calculate the value of a linear gradient portion of the series at the end of the last period in the series (given  $G$ , find  $F$ ).

$$F = G \left[ \frac{(1+i)^N - iN - 1}{i^2} \right] \quad (3.14)$$

This formula along with the corresponding uniform series formula can be used to determine, for example: *“if I open an account and make deposits increasing the amount deposited by \$100 each month, how much will I have in the account after 24 months?”*

- **Converting a linear gradient series to a uniform series:** used to convert a linear gradient series to a uniform series with the same payment timing (given  $G$ , find  $A$ ).

$$A = G \left[ \frac{(1+i)^N - iN - 1}{[i(1+i)^N - 1]} \right] \quad (3.15)$$

After adding this result to the uniform portion of the cash flow series, this formula can answer questions, such as: *“how much would I have to pay if I were to make equal monthly payments on a loan instead of increasing each payment by \$100 each month for the next 12 months?”*

To see how these formulas were derived refer to Section 3.4.7.

Note that, as before,  $i$  is the discount or interest rate per period and  $N$  is the number of periods.

Do not forget that these formulas only apply to the gradient portion of the series. If the series in the problem is split into a gradient series and a uniform series as discussed in this section, then the value of the uniform series must be added to the gradient series value to get the total value of the series.

Let's see some examples of how these formulas are applied.

#### Linear Gradient Series Example #1

*Gary decides to start saving for his newborn twin daughters' post secondary education. He can afford to save \$1000 in the first year, \$1100 in the second year, \$1200 in the third year, and so on, increasing the amount by \$100 each year. The savings account pays 5% interest, compounded annually. How much money will he have in the account after 18 years?*

##### **Solution**

Step 1: Draw the cash flows.

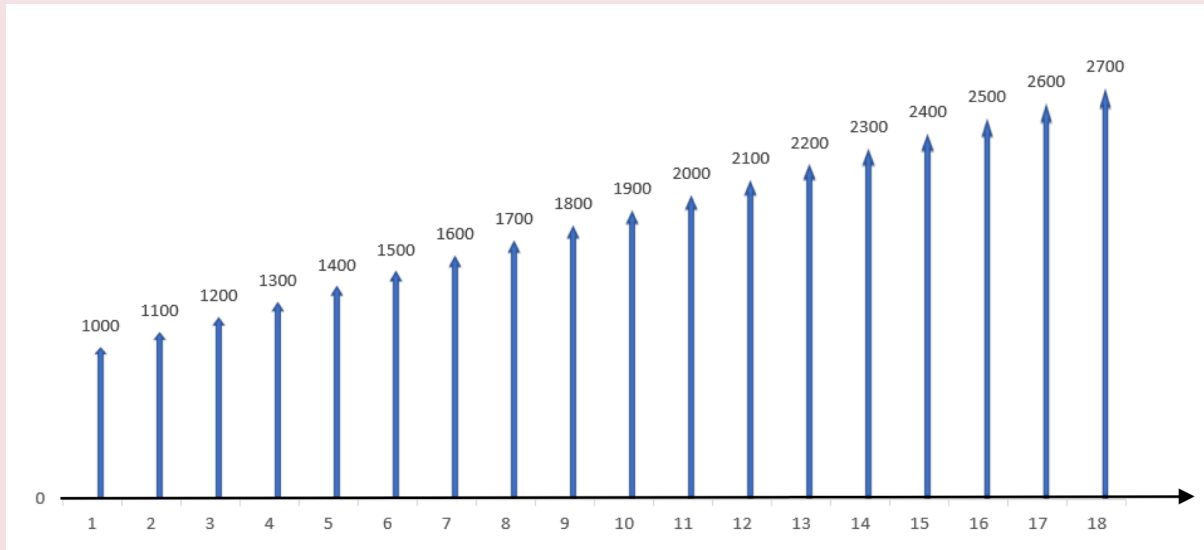


Fig. 3.10 Cash Flow Diagram

Step 2: Split up the cash flow series into two components.

Looking at the cash flow in this problem (Figure 3.10) we can see that it increases by a constant amount (\$100) each, implying a linear gradient. The cash flow in the first period is not zero, so we must split the original cash flow schedule into a linear gradient series and a uniform series.

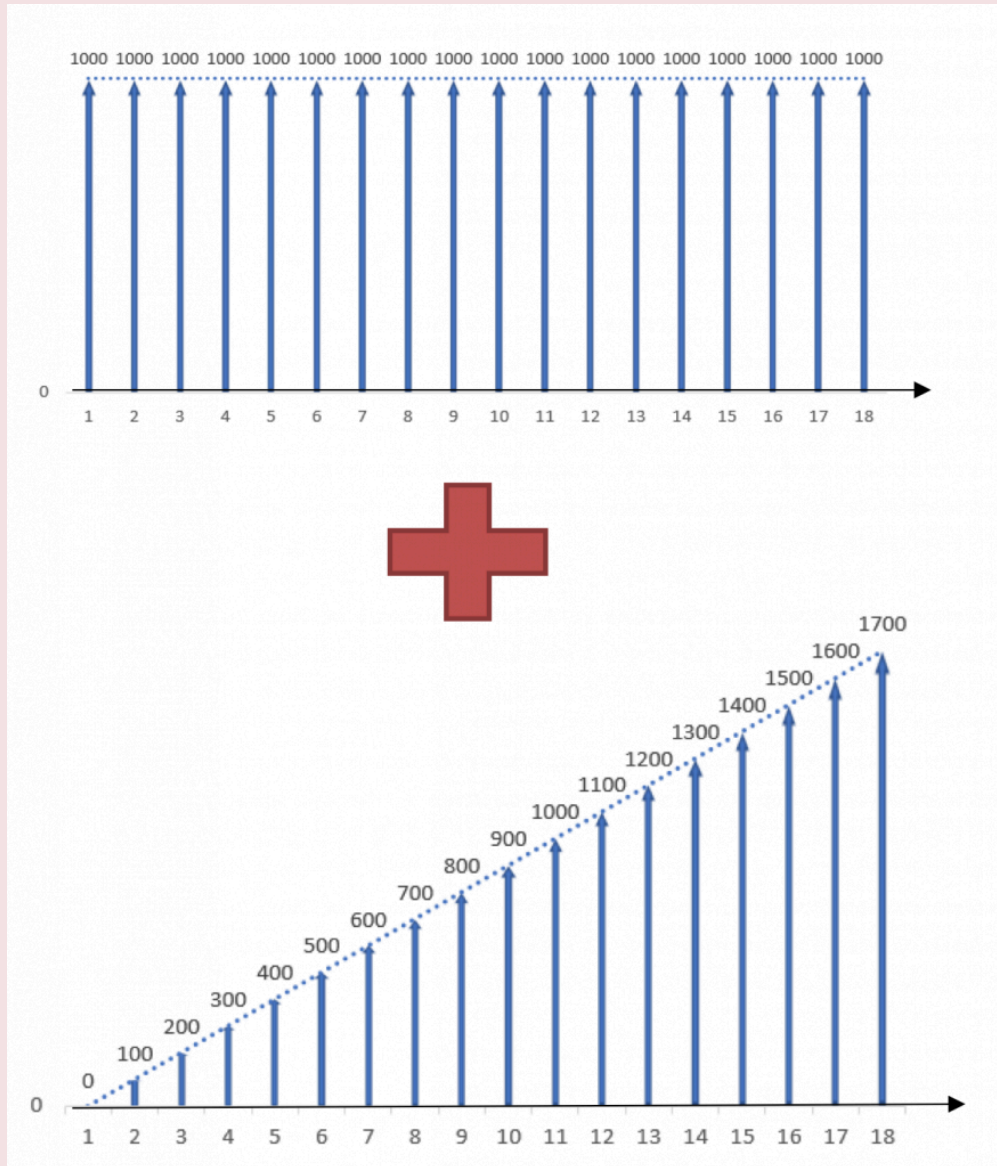


Fig. 3.11 Uniform Series and Linear Gradient Series

The cash flows in the uniform series are equal to the savings in the first period – in this case,  $A = \$1000$ . The gradient is positive, since the cash flows are steadily increasing. For the linear gradient series,  $G = \$100$ .

Step 3: Verify that payments and interest rate have the same time units.

The account pays annual interest and deposits are made annually, so the values have the same time units.

$A = \$1000$  per year

$G = \$100$  per year

$i = 5\%$  per year

$N = 18$  years

Step 3: Calculate the future values of the linear gradient and uniform series using formulas 3.14 and 3.11.

$$F_{grad} = G \left[ \frac{(1+i)^N - iN - 1}{i^2} \right] = \$100 \left[ \frac{(1+0.05)^{18} - (0.05)(18) - 1}{0.05^2} \right] = \$20,264.77$$

$$F_{uniform} = A \left[ \frac{(1+i)^N - 1}{i} \right] = \$1000 \left[ \frac{(1+0.05)^{18} - 1}{0.05} \right] = \$28,132.38$$

Step 4: To calculate the total future value after 18 years, add the future values of the uniform series and the linear gradient series.

$$F_{total} = F_{gradient} + F_{uniform}$$

$$F_{total} = \$20,264.77 + \$28,132.38 = \$48,397.15$$

Thus, after contributing \$33 300 to the account for 18 years, **Gary will have saved \$48 397.15 for his daughters' education.**

### Linear Gradient Series Example #2

*A machine shop is planning to buy a CNC (computer numerical controlled) mill to automate simple steps for large-scale production jobs, saving labour costs. The mill costs \$100 000. Using the mill would result in the following labour cost savings:*

Year	1	2	3	4	5	6
Savings (\$)	28000	26000	24000	22000	20000	18000

*If the company's discount rate is 8%, should the mill be purchased?*

#### Solution

Step 1: Draw cash flows. Note that these values are savings. In situations like this we assume that total cost, which is not stated, remains constant throughout. As a result, these savings also imply that profits increase by the same amount. So we can draw the cash flow diagram as follows:

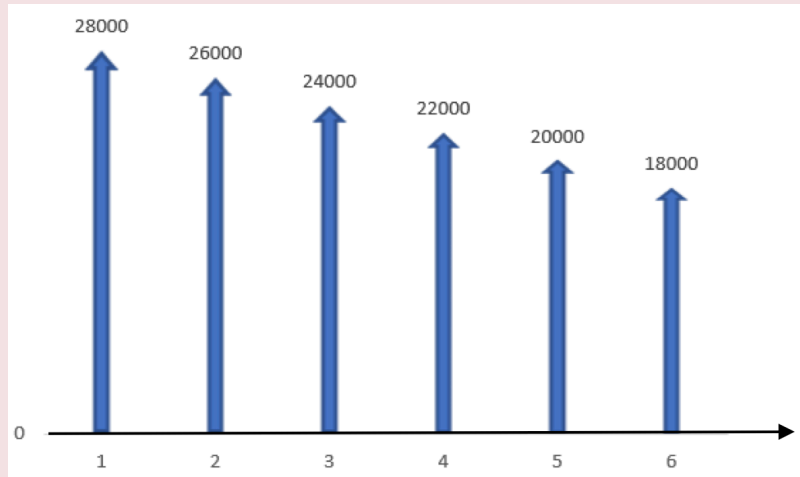


Fig. 3.12 Cash Flow Diagram

Step 2: Separate the cash flows into a uniform series and a linear gradient series.

Looking at the cash flow in this problem (Figure 3.12), we can see that it decreases by a constant amount (\$2000) each year throughout the life of the machine, implying a linear gradient. The cash flow in the first period is not zero, so we must split the original cash flow schedule into a linear gradient series and a uniform series.

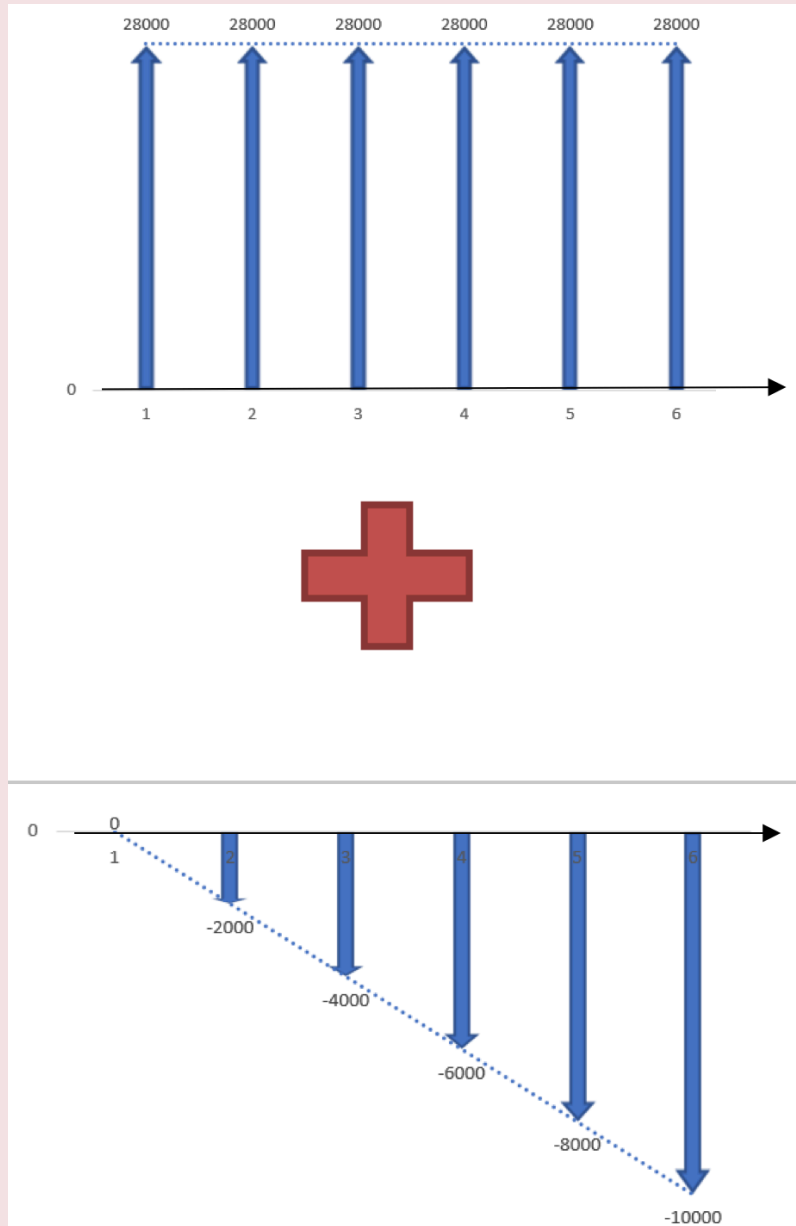


Fig 3.13 Uniform Series and Linear Gradient Series

The cash flows in the uniform series are equal to the savings in the first period, so in this case,  $A = \$28,000$ . The gradient is negative, since the cash flows are steadily decreasing. Thus,  $G = -\$2000$ .

Step 3: Verify that payments and interest rate have the same time units.

Savings in this example are projected on the annual basis and we have an annual discount rate, so the values have the same time units.

$A = \$28,000$  per year

$G = -\$2000$  per year

$i = 8\%$  per year

$N = 6$  years

Step 4 : To determine if the mill is a good investment, we need to find the present value of the overall cash flow series by adding the present values of the two components. We use formulas 3.9 and 3.13 to find present values:

$$P_{overall} = P_{gradient} + P_{uniform}$$

$$P_{overall} = G \left[ \frac{(1+i)^N - iN - 1}{i^2(1+i)^N} \right] + A \left[ \frac{(1+i)^N - 1}{i(1+i)^N} \right]$$

$$P_{overall} = -\$2000 \left[ \frac{(1+0.08)^6 - (0.08)(6) - 1}{0.08^2(1+0.08)^6} \right] + \$28000 \left[ \frac{(1+0.08)^6 - 1}{0.08(1+0.08)^6} \right]$$

$$P_{overall} = -\$21046.55 + \$129440.63 = \$108394.08$$

Therefore, since the present value of the savings, \$108 394.08, is greater than the cost of the mill, \$100 000, purchasing the mill is a good financial investment. Even though the difference of the present value of the savings and the cost of the mill is only \$8394.08, the actual saving from the mill are \$138 000.

### 3.4.4 Geometric Gradient Series

The **geometric gradient series** is a series of cash flows in which each cash flow increases (or decreases) by a constant *percentage* (in contrast to the linear gradient series, where each cash flow increases or decreases by a constant *amount*). The percentage change is called the **geometric gradient**, denoted by  $g$ . The cash flow values are calculated similarly to compound interest: if the cash flow in the first year is  $A_1$ , then in the second year the cash flow will be  $A_1(1 + g)$ , in the third year it is  $A_1(1 + g)^2$ , and so on. This is illustrated in the cash flow diagram below.

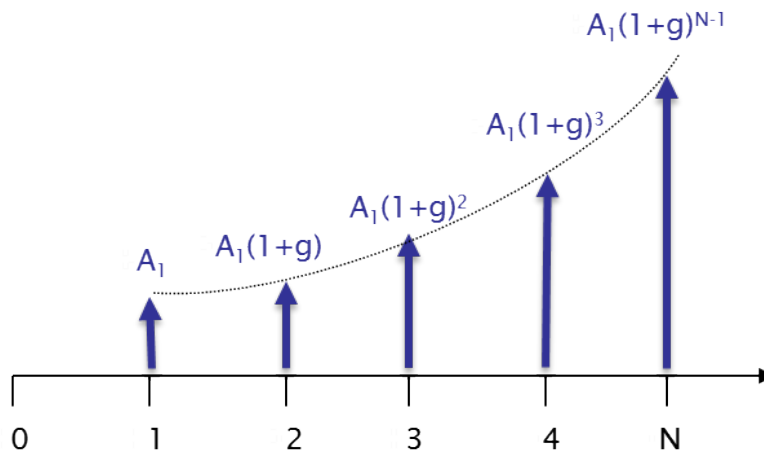


Fig. 3.14 Geometric Gradient Series

As evident from Figure 3.14, just like in the case with compound interest, geometric gradient results in exponential growth. Thus, cash flows are highly sensitive to the gradient value.

The gradient  $g$  can be positive or negative. Positive values of  $g$  lead to a rise in subsequent cash flows, while negative values of  $g$  cause the cash flows to decrease over time. To find  $P$  we need to “move” all of the cash flows to period 0.

The first step in solving the gradient portion of the series is to check whether the geometric gradient ( $g$ ) is equal to the discount rate ( $i$ ). If they are not equal, the formulas are:

- When  $i \neq g$ :

$$P = \frac{A_1}{i-g} \left[ 1 - \left( \frac{1+g}{1+i} \right)^N \right] \quad (3.16)$$

$$F = \frac{A_1}{i-g} \left[ (1+i)^N - (1+g)^N \right] \quad (3.17)$$

If they are equal, the denominator of the first term on the right-hand side of each equation will equal 0 and cannot be solved. Thus, the following formulas need to be used instead:

- When  $i = g$

$$P = A_1 \left( \frac{N}{1+i} \right) \quad (3.18)$$

$$F = N A_1 (1+i)^{N-1} \quad (3.19)$$

These formulas are used to answer questions such as: “*what is the present (or future) value of a project that is expected to yield \$1000 in profit this month, and the profit is expected to increase by 5% every following month for 11 months?*”

**Important note: be careful not to confuse  $i$  and  $g$ .** The geometric gradient,  $g$ , is the rate at which the cash flows increase or decrease in subsequent periods, which reflects changes in the nominal value of the cash flows. The discount rate,  $i$ , is used to account for the time value of money.

When using the geometric gradient series formulas:

- $N$  is the number of interest periods, including the initial period where the gradient is equal to zero.
- The discount rate,  $i$ , and the gradient,  $g$ , must be specified. The MARR (which will be covered in chapter 5) is often used as the discount rate.
- The cash flow is **NOT** divided into uniform and gradient components. It can be solved directly using the above formulas.

Let’s see some example problems where geometric gradient series formulas are used.



### Geometric Gradient Series Example

Brad, a civil engineer, is coordinating a project to build a new stadium in Regina. The cost of stadium construction is estimated to be \$278 million. [6] Part of the sum will be collected in the form of grants from the Provincial Government, the City of Regina and the Saskatchewan Roughriders. The rest, which amounts to \$100 million, will be loaned by the Province for a period of 30 years at 3.5% interest, compounded annually. The loan is to be repaid on a monthly basis with equal payments throughout the entire term.

The City will use ticket sales revenues for some of the events held at the stadium to make the loan payments. In the first year, the revenues are expected to be equal to the total loan payments for the year. The project management team expects these revenues to increase by 5% each year.

Does the present value of the total revenue from ticket sales exceed the value of the loan? (Assume 6% discount rate)

#### Solution

First, we need to determine the monthly loan payments, so that we can determine how much is to be repaid in the first year, which also tells us the expected ticket revenues in the first year. Thus, we will solve this problem in two stages: calculating the equal payments of the loan and calculating the total revenues from ticket sales.

#### Part 1:

Step 1: To calculate the equal payment value, we use the payments-from-present value formula 3.10. We have the following information:

$$N = 30,$$

$$i = 3.5\% = 0.035, \text{ and}$$

$$P_{\text{loan}} = \$100 \text{ million.}$$

To solve for A:

$$A = P_{\text{loan}} \left[ \frac{i(1+i)^N}{(1+i)^N - 1} \right] + \$100 \text{million} \left[ \frac{0.035 * (1+0.035)^{30}}{(1+0.035)^{30} - 1} \right] = \$5.44 \text{million}$$

So, the annual loan payments are \$5.44 million.

#### Part 2:

Step 3: Now that we know the value of the yearly loan payment, we can conclude that ticket sales in the first year are expected to be \$5.44 million. Since, project management team expects the revenues to increase by 5% each year, this implies a geometric gradient series with the first cash flow  $A=A_1=\$5.44$  million.

To find the present value of the total revenue from ticket sales at the end of 30 years, the present value formula for geometric series is used. Note that, so we use formula 3.16 to calculate future value of the total revenue. Thus, we have:

$$N = 30,$$

$$i = 6\% = 0.06$$

$$g = 5\% = 0.05, \text{ and}$$

$$A=A_1=\$5.44 \text{ million}$$

$$P_{\text{totalrevenue}} = \frac{A_1}{i-g} \left[ 1 - \left( \frac{1+g}{1+i} \right)^N \right] = \frac{\$5.44 \text{million}}{0.06-0.05} \left[ 1 - \left( \frac{1+0.05}{1+0.06} \right)^{30} \right] = \$544 \text{million} [1 - 0.075] = \$136 \text{million}$$

So, the present value of the total revenue from ticket sales exceeds the value of the loan by \$36 million.

### 3.4.5 Complex Cash Flows

The purpose of cash-flow equations is to simplify calculations required for cash flow analysis. It is usually much more convenient to group several cash flows together for one calculation rather than to discount each cash flow separately.

To apply the appropriate formulas, it is important to identify the type of cash flow series. However, in practice, it is not always apparent what type of series we are dealing with. Cash flows can be seemingly “random” or change drastically over the analysis period. We call these **complex cash flows**. To tackle complex cash flows problems, the trick is to seek out patterns in the cash flow schedules and “deconstruct” them into different cash flow series, which may allow us to use the cash-flow equations.

The example below demonstrates how complex cash flows can be approached.

#### A complex cash flow example

*Given the schedule of cash flows in Figure 3.15, find the total present value of the series at year 0 using a discount rate of 8%.*

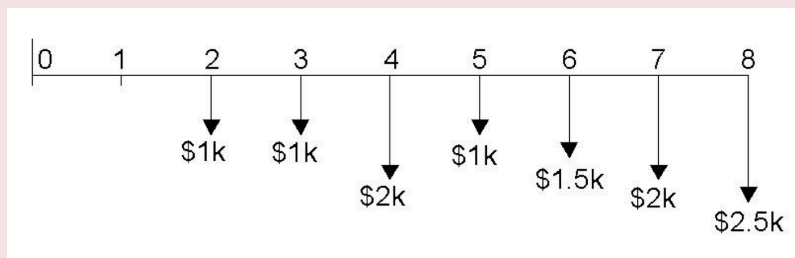


Fig. 3.15 Schedule of Cash Flows

To find the present value of this cash flow series, we must “move” each amount to year 0. One way to do so is to discount each cash flow one-by-one using the present-value formula 3.8 for single cash flows and sum them. However, it may be less time-consuming to recognize that the individual flows can be grouped into separate series.

Step 1: Examine the cash flow diagram for patterns. One set of patterns is as follows:

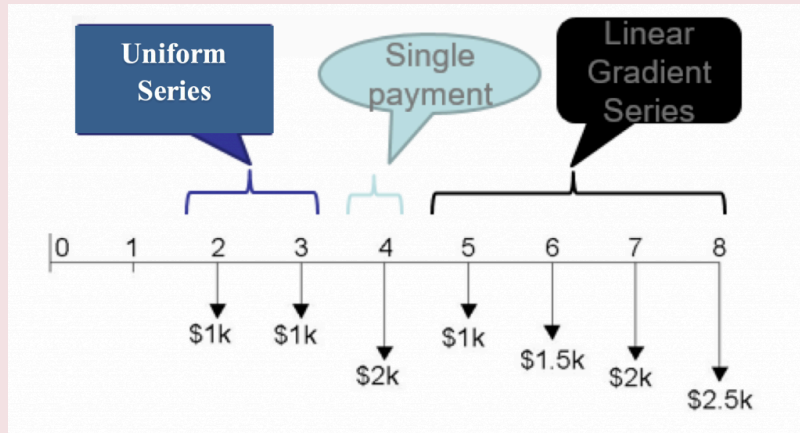


Fig. 3.16 Grouping of the Cash Flows

Note that this is not the only possible way to segment the cash flows, but this is the segmentation we will use to solve the problem.

Step 2: Now to calculate the total present value we will begin by moving each of the three series to period 0.

#### Segment A: The uniform series

The uniform series begins in year 2, so we calculate its “present” value from the uniform series formula 3.9 to get the series’ value in year 1, which we will call  $P_1$ . Then, we move  $P_1$  to year zero by discounting it over one year (see Figure 3.17).

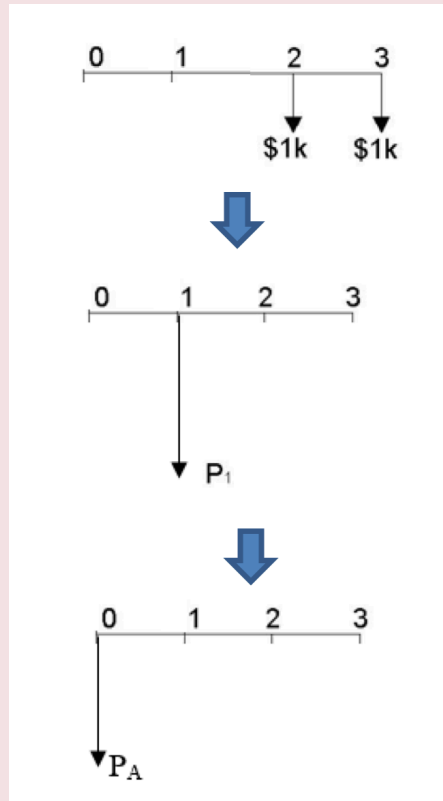


Fig. 3.17 Discounting of a Uniform Series

$$P_1 = A \left[ \frac{(1+i)^N - 1}{i(1+i)^N} \right] = -\$1000 \left[ \frac{(1+0.08)^2 - 1}{0.08(1+0.08)^2} \right] = -\$1783.26$$

$$P = \frac{P_1}{(1+i)} = \frac{-\$1783.26}{(1+0.08)} = -\$1651.17$$

**Segment B: The single payment**

To determine the present value of the single payment in year 4 we discount it 4 years to bring it to the present, as shown in Figure 3.18:

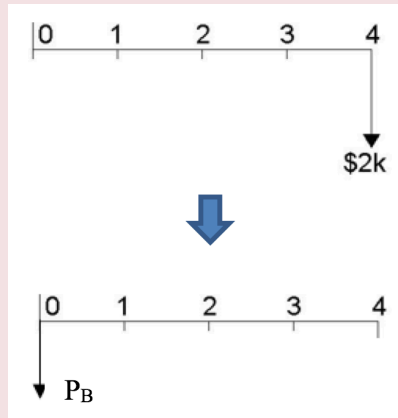


Fig. 3.18 Discounting of a Single Payment

$$P_B = \frac{F_4}{(1+i)^N} = \frac{=\$2000}{(1+0.08)^4} = -\$1470.06$$

### Segment C: The linear gradient series

Recall that linear gradient series must be split into two series: a uniform series and a gradient series. In this example, the first cash flow of the linear gradient series is -\$1000, so the uniform series will have equal payments,  $A = -\$1000$ . Each consecutive outflow increases by \$500, so the gradient,  $G = -\$500$  (recall, we treat outflows as negative values). The first cash flow of the segment occurs at year 5, so when we use equations to calculate the present value, the resulting present value is actually in year 4. So we must remember to “move” it to year 0, as shown in Figure 3.19:

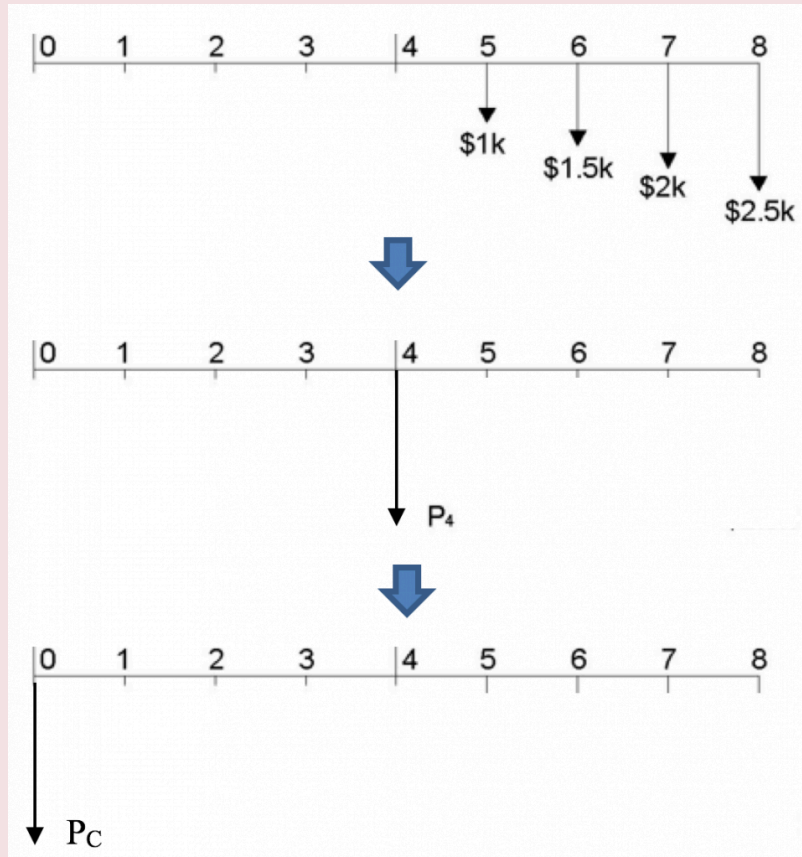


Fig. 3.19 Discounting of a Linear Gradient Series

$$\begin{aligned}
 P_4 &= P_{uniform} + P_{gradient} = A \left[ \frac{(1+i)^N - 1}{i(1+i)^N} \right] + G \left[ \frac{(1+i)^N - iN - 1}{i^2(1+i)^N} \right] \\
 &= -\$1000 \left[ \frac{(1+0.08)^4 - 1}{0.08(1+0.08)^4} \right] - \$500 \left[ \frac{(1+0.08)^4 - (0.08)(4) - 1}{0.08^2(1+0.08)^4} \right] \\
 &= -\$3312.13 - \$2325.05 = -\$5637.18
 \end{aligned}$$

We now “move” the year 4 present value to year 0:

$$P_C = \frac{P_4}{(1+i)^N} = \frac{-\$5637.18}{(1+0.08)^4} = -\$4143.50$$

Step 3: To obtain the present value of the complete series we sum the present values of each segment:

$$\text{Total } P = P_A + P_B + P_C$$

$$\text{Total } P = (-\$1651.17) + (-\$1470.06) + (-\$4143.50) = -\$7264.73$$

So, the present value of the complete series (Total P) is -\$7264.73.

### 3.4.6 Summary of Equations

The summary of equations of economic equivalence is in Table 3.4 below.

Formula			Factor Notation
Present Value Formula for Single Cash Flows	$P = \frac{F}{(1+i)^N}$	P = present value, \$ F = future value, \$ i = interest rate, % N = number of periods	(P/F, i, N)
Future Value Formula for Single Cash Flows	$F = P(1+i)^N$		(F/P, i, N)
Present Value from Payment Formula for Uniform Series	$P = A \left[ \frac{(1+i)^N - 1}{i(1+i)^N} \right]$		(P/A, i, N)
Payment from Present Value Formula for Uniform Series	$A = P \left[ \frac{i(1+i)^N}{(1+i)^N - 1} \right]$		(A/P, i, N)
Future Value from Payment Formula for Uniform Series	$F = A \left[ \frac{(1+i)^N - 1}{i} \right]$		(F/A, i, N)
Payment from Future Value Formula for Uniform Series	$A = F \left[ \frac{i}{(1+i)^N - 1} \right]$		(A/F, i, N)
Present Value of Linear Gradient Series	$P = G \left[ \frac{(1+i)^N - iN - 1}{i^2(1+i)^N} \right]$	G = Linear Gradient	(P/G, i, N)
Future Value of Linear Gradient Series	$F = G \left[ \frac{(1+i)^N - iN - 1}{i^2} \right]$		(F/G, i, N)
Linear Gradient to Uniform Series Conversion	$A = G \left[ \frac{(1+i)^N - iN - 1}{i[(1+i)^N - 1]} \right]$		(A/G, i, N)
Present Value of Geometric Gradient Series, when $i \neq g$	$P = \frac{A_1}{i-g} \left[ 1 - \left( \frac{1+g}{1+i} \right)^N \right]$	g = Geometric Gradient	(P/A <sub>1</sub> , g, i, N)
Future Value of Geometric Gradient Series, when $i \neq g$	$F = \frac{A_1}{i-g} [(1+i)^N - (1+g)^N]$		(F/A <sub>1</sub> , g, i, N)
Present Value of Geometric Series, when $i = g$	$P = A_1 \left( \frac{N}{1+i} \right)$	$i = g$	(P/A <sub>1</sub> , i, N)
Future Value of Geometric Series, when $i = g$	$F = NA_1 (1+i)^{N-1}$		(F/A <sub>1</sub> , i, N)

Table 3.4 Equations of Economic Equivalence Formulas

The final column of this table shows what is called factor notation – a short-hand way of expressing the equations. For example, let's take a look at the present value for the single cash flows formula (row 1).

$$P = \frac{F}{(1+i)^N}$$

So,  $P/F = \frac{1}{(1+i)^N}$ . The right-hand side of this equation is called a factor. In factor notation this is shown as

$$(P/F, i, N)$$

This factor is read as “P divided by F, given i and N”.

So, to represent the entire equation, we write

$$P = F(P/F, i, N)$$

Similarly, using factor notation, the future this factor is expressed as (F/P, i, N), which is read as “F divided by P, given i and N”. The future value formula, thus, becomes:

$$F = P(F/P, i, N)$$

As you browse through the formulas in the table, you will notice that each equation requires multiplication involving a value of a cash flow (present value, future value, annuity, linear gradient) and a factor which depends on  $i$  and  $N$  (also  $g$  when dealing with geometric gradient series). To simplify calculations, factor values for different  $i$  and  $N$  have been tabulated in so-called *compound interest factor tables* for equations of economic equivalence. These tables allow to substitute already calculated factor values given interest-rate-and-number-of-periods scenarios in the appropriate formulas, hence decreasing the amount of necessary calculations.

### 3.4.7 Derivations

The following sections provides derivations for the formulas used in Section 3.4. These derivations are provided to help you understand how the equations were developed. The ability to independently arrive at these derivations is not essential to being able to use them correctly or have a solid understanding of economic equivalence.

#### Single cash flows formulas:

- **Future Value formula for single cash flows:**

$$F = P(1 + i)^N \quad (3.6)$$



1. Recall that the future value is equal to the present value and accumulated interest.
2. The interest earned in period 1, is equal to  $I_1 = Pi$ . So, the future value in period 1 becomes
3. In period 2, interest is applied to the ending balance in year 1, which is  $F_1: I_2 = F_1 i$ . Thus, the future value in period 2 is
4. Since, from equation (2),  $F_1 = P(1 + i)$ , we can substitute this into equation (3), yielding
5. This simplifies to
6. In period 3, interest is now applied to the present value in period 2, which is  $F_2: I_3 = F_2 i$ . Hence, the future value in period 3 becomes
7. Since, from equation (5)  $F_2 = P(1 + i)^2$ , substituting the right-hand side from of the equation into equation (6) yields
8. Simplifying, we get
9. Continuing for N periods using the same steps, we get the future value formula for single cash flows for N periods and interest rate  $i$ .

- **Present value formula for single cash flows** (present value with compound interest formula):

$$P = \frac{F}{(1+i)^N} \quad (3.8)$$

The present value formula can be obtained from the future value formula for single cash flows 3.6:

1. Recall the future value formula 3.6
2. To obtain P, we divide both sides by  $(1 + i)^N$ . We now have the present value formula for single cash flows.

## Uniform Series formulas:

- **Present-value-from-payment formula:**

$$P = A \left[ \frac{(1+i)^N - 1}{i(1+i)^N} \right] \quad (3.9)$$

---

1. The present value is the sum of all discounted cash flows in the series. In a uniform series, each cash flow has a value of  $A$ .

---

2. Now we multiply both sides of the equation by  $(1 + i)$ . On the right-hand side, all denominators' exponents decrease by 1.

---

3. We need to get the right-hand side in equation (2) from equation (1). To do that, first, we move  $\frac{A}{(1+i)^N}$  to the left-hand side in equation (1).

---

4. We then add  $A$  to both sides of equation (3), which yields equation (4).

---

5. Now, we relate equations (1) and (4) to obtain an equation without series (5).

$$(1 + i)P = P + A$$

---

6. We now rearrange the terms in equation (5) to solve for  $P$ .

---

7. Multiplying the terms inside the brackets in equation (6) by  $i$  we obtain the present-value-from-payment formula (7).

---

This formula combined with the present-value formula for single cash flows 3.8 is used to derive the three remaining formulas: the payment-from-present value formula 3.10, the future-value-from-payments formula 3.11 and the payment from future value formula 3.12.

- **Payment-from-present-value formula:**

$$A = P \left[ \frac{i(1+i)^N}{(1+i)^N - 1} \right] \quad (3.10)$$

The payment-from-present-value formula 3.10 and the present-value-from-payment formula 3.9 are reciprocals of each other:

---

1. Recall the present-value-from-payment formula 3.9

---

2. To obtain  $A$ , we divide both sides by  $\left[ \frac{(1+i)^N - 1}{i(1+i)^N} \right]$

---

3. Rearranging equation (2) we get the payment-from-present-value formula

- **Future-value-from-payment formula:**

$$F = A \left[ \frac{(1+i)^N - 1}{i} \right] \quad (3.11)$$

This formula can be obtained using the payment-from-present value formula 3.9 and the future value for single cash flows formula 3.6:

---

1. Recall the present-value-from-payment formula 3.9

---

2. Next, recall the present value formula for single cash flows 3.8

---

3. Substituting  $\frac{F}{(1+i)^N}$  from equation (2) in equation (3) yields

---

4. Now we multiply both sides of equation (3) by  $(1+i)^N$

---

5. This, in turn, cancels out  $(1+i)^N$  in the denominator in the term in square brackets on the right-hand side in equation (4). We get the future-value-from-payment formula

---

• **Payment-from-future-value formula:**

$$A = F \left[ \frac{i}{(1+i)^N - 1} \right] \quad (3.12)$$

The payment-from-future-value formula 3.12 and the future-value-from-payment formula 3.11 are reciprocals of each other:

---

1. Recall the future-value-from-payment formula 3.11

---

2. Solving for A, we divide both sides by  $\left[ \frac{(1+i)^N - 1}{i} \right]$

---

3. Rearranging equation (2) we get the payment-from-future-value formula

---

**Linear Gradient Series Formulas:**

These formulas apply to the gradient series with linear gradient G as illustrated in Figure 3.20.

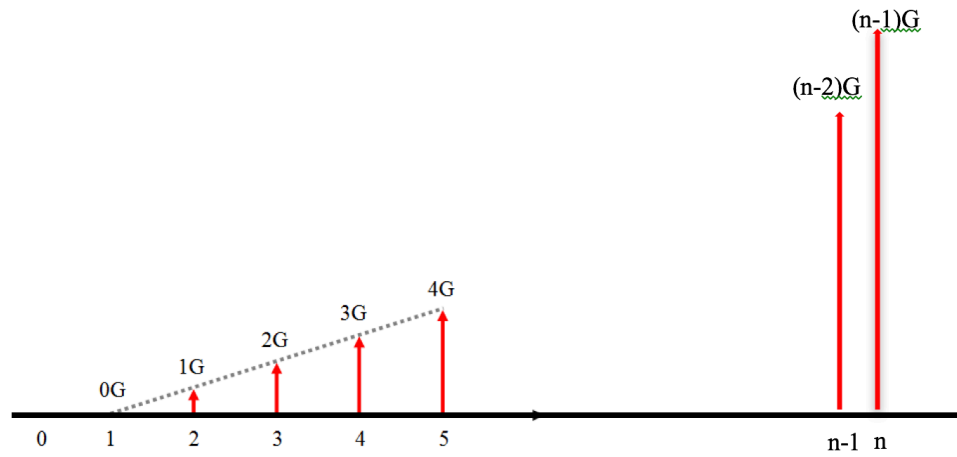


Fig. 3.20 Linear Gradient Series

- **Future value of a linear gradient series formula:**

$$F = G \left[ \frac{(1+i)^N - iN - 1}{i^2} \right] \quad (3.14)$$

1. Future value of the linear gradient series can be thought of as a sum of future values of individual cash flows in the series:  $0G, 1G, 2G \dots (n-2)G, (n-1)G$ . Let  $i$  be the discount rate for discounting cash flows. The future value of the gradient series can thus be expressed as

$$F = G(1+i)^{N-2} + 2G(1+i)^{N-3} + \dots + (n-1)G(1+i)$$

2. Now we multiply both sides of the equation (1) by  $(1+i)$  and factor out  $G$ . We will need this equation after the next step.

$$F(1+i) = G[(1+i)^{N-1} + 2(1+i)^{N-2} + \dots + (n-1)(1+i)^2]$$

3. We go back to equation (1) again and now we only factor out  $G$

$$F = G[(1+i)^{N-2} + 2(1+i)^{N-3} + \dots + (n-1)(1+i)]$$

4. Next, we subtract equation (3) from equation (2).

$$F(1+i) - F = G[(1+i)^{N-1} + 2(1+i)^{N-2} + \dots + (n-1)(1+i)^2 - (1+i)^{N-2} - 2(1+i)^{N-3} - \dots - (n-1)(1+i)]$$

4.1 We rearrange the terms on the right-hand side of the equation by writing the terms that include  $(1+i)$  with the same power from equation (2) and (3) next to each other. This allows us to subtract and rearrange them next.

$$\rightarrow F + iF - F = G[(1+i)^{N-1} + (1+i)^{N-2} - (1+i)^{N-2} - (1+i)^{N-3} + (1+i)^{N-3} - (1+i)^{N-4} + \dots + (1+i)^2 - (1+i)]$$

4.2 We leave the first term in the brackets; subtract the next two, obtaining  $(2-1)(1+i)^{N-2}$  as a result; we factor out  $(1+i)$  from the two terms after the ellipsis getting  $(1+i)(n-1-n+2)$ ; and leave the last term in the equation.

$$\rightarrow iF = G[(1+i)^{N-1} + (2-1)(1+i)^{N-2} + \dots + (1+i)(n-1-n+2)]$$

4.3 Further opening the brackets and simplifying the equation we get equation (4)

$$\rightarrow iF = G[(1+i)^{N-1} + (1+i)^{N-2} + \dots + (1+i)]$$

5. Now we carry  $n$  out of the brackets in the right-hand side of equation (5)

$$\rightarrow iF = G[(1+i)^{N-1} + (1+i)^{N-2} + \dots + (1+i)]$$

6. Notice that the expression in the square brackets is geometric series, where the first term of the series is  $(1+i)^{N-1}$  and each subsequent term is multiplied by  $\frac{1}{(1+i)}$ . We can find the sum of the geometric series using the geometric series formula  $\sum_{K=0}^{n-1} (ar^K) = a\left(\frac{1-r^n}{1-r}\right)$ , where  $a = (1+i)^{N-1}$  and  $r = \frac{1}{(1+i)}$ .

We write down the geometric series summation formula and substitute the values from the series in equation (5).

After some algebraic manipulation, we obtain the sum of the geometric series, which is expressed by equation (6).

7. Now we substitute equation (6) in equation (5) to get rid of the series in the equation

8. Next, we solve for F by dividing both sides of equation (7) by i and rearranging the terms on the right-hand side of the equation, obtaining the future value of a linear gradient series formula

- **Present value of a linear gradient series formula:**

$$P = G \left[ \frac{(1+i)^N - iN - 1}{i^2(1+i)^N} \right] \quad (3.13)$$

The present value of a linear gradient series formula 3.13 can be obtained from the present value formula for single cash flows 3.8 and future value of a linear gradient series formula 3.14:

---

1. Recall the future value of a linear gradient series formula 3.14

---

2. To obtain P, we divide both sides by  $\frac{F}{(1+i)^N}$  from the present value formula for single cash flows 3.8  $P = \frac{F}{(1+i)^N}$

---

3. Multiplying the two fractions, we get the present value of a linear gradient series formula

- **Converting a linear gradient series to a uniform series formula:**

$$A = G \left[ \frac{(1+i)^N - iN - 1}{i[(1+i)^N - 1]} \right] \quad (3.15)$$

This formula can be derived from the present value of the linear gradient series formula 3.13 and the present-value-from-payment formula 3.9:

---

1. Recall the present value of a linear gradient series formula 3.13

---

2. Also recall the present-value-from-payment formula 3.9

---

3. We now substitute the right-hand side from equation (2) into the left-hand side of equation (1)

---

4. To solve for A, we divide both sides of equation (3) by  $\frac{(1+i)^N - 1}{i(1+i)^N}$

---

5. Rearranging and simplifying equation (4) we get the formula for converting linear gradient series to a uniform series

**Geometric Gradient Series Formulas:**

When  $i \neq g$

- **Present value for geometric gradient series formula:**

$$P = \frac{A_1}{i-g} \left[ 1 - \left( \frac{1+g}{1+i} \right)^N \right] \quad (3.16)$$

1. The present value of the geometric gradient series can be expressed as a sum of discounted values of individual cash flows in the series  $A_1, A_1(1+g), A_1(1+g)^2, \dots, A_1(1+g)^{N-1}$ . Let  $i$  be the discount rate for discounting cash flows. The present value of the gradient series can thus be expressed as

$$P = \frac{A_1}{(1+i)^1} + \frac{A_1(1+g)^1}{(1+i)^2} + \frac{A_1(1+g)^2}{(1+i)^3} + \dots$$

2. We now factor out  $A_1$  in equation (1). We will need this equation after the next step.

$$P = A_1 \left[ \frac{1}{(1+i)^1} + \frac{(1+g)^1}{(1+i)^2} + \frac{(1+g)^2}{(1+i)^3} + \dots \right]$$

3. Now we multiply both sides of the equation (2) by  $\frac{1+g}{1+i}$ .

$$P \frac{(1+g)}{(1+i)} = A_1 \left[ \frac{(1+g)}{(1+i)^2} + \frac{(1+g)^2}{(1+i)^3} + \frac{(1+g)^3}{(1+i)^4} + \dots \right]$$

4. Next, we subtract equation (2) from equation (3).

$$P \frac{(1+g)}{(1+i)} - P = A_1 \left[ \frac{(1+g)}{(1+i)^2} + \frac{(1+g)^2}{(1+i)^3} + \frac{(1+g)^3}{(1+i)^4} + \dots + \frac{(1+g)^{N-1}}{(1+i)^N} - \frac{1}{(1+i)^1} - \frac{(1+g)^1}{(1+i)^2} - \frac{(1+g)^2}{(1+i)^3} - \dots - \frac{(1+g)^{N-1}}{(1+i)^N} \right]$$

On the right-hand side of the equation, we rearrange the terms in brackets, placing same terms from equations (2) and (3) next to one another.

Notice, that other than the first and the last terms in brackets, all the other terms will cancel out. So, rearranging the equation and canceling out the terms we get equation (4)

$$\begin{aligned} &\rightarrow P \left[ \frac{(1+g)}{(1+i)} - 1 \right] = A_1 \left[ -\frac{1}{(1+i)^1} + \frac{(1+g)^{N-1}}{(1+i)^N} + \frac{(1+g)^N}{(1+i)^{N+1}} \right] \\ &\rightarrow P \left[ \frac{1+g}{1+i} - 1 \right] = A_1 \left[ -\frac{1}{(1+i)} + \frac{(1+g)^N}{(1+i)^{N+1}} \right] \end{aligned}$$

5. Now we solve for  $P$ . First, we divide both sides of the equation by  $\left[ \frac{1+g}{1+i} - 1 \right]$

$$= \frac{1+g-1-i}{1+i} = \frac{g-i}{1+i}$$

$$\begin{aligned} &P \left[ \frac{1+g}{1+i} - 1 \right] = A_1 \left[ -\frac{1}{(1+i)} + \frac{(1+g)^N}{(1+i)^{N+1}} \right] \\ &\rightarrow P = \frac{A_1 \left[ -\frac{1}{(1+i)} + \frac{(1+g)^N}{(1+i)^{N+1}} \right]}{\frac{g-i}{1+i}} \end{aligned}$$

6. Canceling out  $(1+i)$  in equation (5) and further rearranging and simplifying the equation, we get the present value for geometric gradient series formula for the case when  $i \neq g$ .

$$\begin{aligned} P &= \frac{A_1 \left[ -\frac{1}{(1+i)} + \frac{(1+g)^N}{(1+i)^{N+1}} \right]}{\frac{g-i}{1+i}} \\ &\rightarrow P = \frac{A_1 \left[ -1 + \frac{(1+g)^N}{(1+i)^N} \right]}{g-i} \\ &\rightarrow P = \frac{A_1 \left[ \frac{(1+g)^N}{(1+i)^N} - 1 \right]}{g-i} \end{aligned}$$

- **Future value for geometric gradient series formula:**

$$F = \frac{A_1}{i-g}[(1+i)^N - (1+g)^N] \quad (3.17)$$

This formula can be derived from the present value for geometric gradient series formula 3.16 and the future-value-from-payment formula 3.6:

1. Recall the present value for geometric gradient series formula 3.16	$P = \frac{A_1}{i-g} [$
2. To obtain F, we multiply both sides by $(1+i)^N$ from the future value formula for single cash flows 3.6 $F = (1+i)^N P$	$F = \frac{A_1}{i-g} [1 - ($
3. Multiplying the terms in equation (2) and simplifying the equation, we get the future value for geometric gradient series formula for the case when $i \neq g$ .	$F = \frac{A_1}{i-g} [(1 +$

When  $i = g$

When discounting the cash flows in the case where the discount (interest) rate is equal to the geometric gradient,  $i = g$ , all the cash flows have the same present value. This is because the cash flows are increasing at the same rate as they are being discounted, thus maintaining the purchasing power of the cash flow. Therefore, the formulas for this case will be different, although the approach used to derive these formulas is the same.

• **Present value for geometric gradient series formula:**

$$P = A_1 \left( \frac{N}{1+i} \right) \quad (3.18)$$

1. Like in the case where $i = g$ , the present value of the geometric gradient series can be expressed as a sum of discounted values of individual cash flows in the series. Now that the gradient and the discount rates are equal ( $i = g$ ), these cash flows are $A_1, A_1(1+i), A_1(1+i)^2 \dots A_1(1+i)^{N-1}$ . The present value of the gradient series can thus be expressed as	$P = \frac{A_1}{i-g} [$
2. To obtain F, we multiply both sides by $(1+i)^N$ from the future value formula for single cash flows 3.6 $F = (1+i)^N P$	$F = \frac{A_1}{i-g} [1 -$
3. Multiplying the terms in equation (2) and simplifying the equation, we get the future value for geometric gradient series formula for the case when $i \neq g$ .	$F = \frac{A_1}{i-g} [(1 +$

When  $i = g$

When discounting the cash flows in the case where the discount (interest) rate is equal to the geometric gradient, , all the cash flows have the same present value. This is because the cash flows are increasing at the same rate as they are being discounted, thus maintaining the purchasing power of the cash flow. Therefore, the formulas for this case will be different, although the approach used to derive these formulas is the same.

• **Present value for geometric gradient series formula:**

$$P = A_1 \left( \frac{N}{1+i} \right) \quad (3.18)$$



1. Like in the case where  $i \neq g$ , the present value of the geometric gradient series can be expressed as a sum of discounted values of individual cash flows in the series. Now that the gradient and the discount rates are equal ( $i = g$ ), these cash flows are  $A_1, A_1(1+i), A_1(1+i)^2 \dots A_1(1+i)^{N-1}$ . The present value of the gradient series can thus be expressed as

$$P = \frac{A_1}{(1+i)^1} + \frac{A_1(1+i)}{(1+i)^2} + \frac{A_1(1+i)^2}{(1+i)^3} + \dots + \frac{A_1(1+i)^{N-1}}{(1+i)^N}$$

2. We now factor out  $A_1$  in equation (1).

$$P = A_1 \left[ \frac{1}{(1+i)^1} + \frac{1+i}{(1+i)^2} + \frac{(1+i)^2}{(1+i)^3} + \dots + \frac{(1+i)^{N-1}}{(1+i)^N} \right]$$

3. On the right-hand side of equation (2) we have  $N$  terms, each being  $\frac{1}{(1+i)^1}$ . The sum of these terms is  $\frac{N}{(1+i)^1}$ . Substituting this expression into equation (2) yields the

• **Future value for geometric gradient series formula:**

$$F = NA_1(1+i)^{N-1} \quad (3.19)$$

This formula can be derived from the present value for geometric gradient series formula 3.18 and the future-value-from-payment formula 3.6:

1. Recall the present value for geometric gradient series formula 3.18

$$P = \frac{A_1}{i} \left[ 1 - \frac{1}{(1+i)^N} \right]$$

2. To obtain  $F$ , we multiply both sides by  $(1+i)^N$  from the future value formula for single cash flows 3.6  $F = P(1+i)^N$

$$F = A_1 \left[ 1 - \frac{1}{(1+i)^N} \right] (1+i)^N$$

3. Multiplying the terms in equation (2) and simplifying the equation, we get the future value for geometric gradient series formula for the case when  $i = g$ .

$$F = A_1 \left[ 1 - \frac{1}{(1+i)^N} \right] (1+i)^N$$

[6] Note amounts in this problem are indicated in US dollars, so the amount in Canadian dollars will differ. However, this is irrelevant to the problem

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## 3.5 Discussion Questions

### Discussion and Review Questions

1. What is the difference between simple and compound interest?
2. Briefly explain what inflation is, and why it affects financial decisions.
3. Briefly explain the concept of the time value of money. How does the time value of money influence financial decision-making?
4. How do you determine if two proposals are economically equivalent?
5. List the common types of cash flow series and give an example of each.

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## 3.6 Glossary of Terms

### Glossary of Terms

- **Annuity (A):** An equal payment made every period.
- **Cash flow diagram:** A diagram showing the timing and value of cash inflows and outflows, as viewed from one party's perspective.
- **Discount rate:** The rate used to “move” cash flows to other time periods to account for the time value of money.
- **Discounting:** The process of “moving” cash flows to other time periods while retaining economic equivalence.
- **End-of-period convention:** all cash flows that occur within an interest period are added together and treated as if they occurred on the last day of the interest period. This simplifies calculations, but may introduce some error.
- **Future value (F):** The nominal value of a cash flow at some future date.
- **Geometric gradient (g):** A percentage amount by which each consecutive cash flow increases (or decreases) in a geometric gradient series.
- **Geometric gradient series:** A series of cash flows in which each consecutive cash flow is larger or smaller than the previous by a percentage.
- **Inflation:** An increase in the price level of goods and services in an economy over time. Negative inflation (i.e. decrease in the price level over time) is called **deflation**.
- **Interest:** the cost of borrowing money. It is the amount of money that is paid in addition to the amount borrowed, loaned or invested.
- **Interest rate:** a percentage rate applied on the principal, which is used to calculate the amount of interest generated in an **interest period** (the frequency that interest is calculated, e.g. monthly, bi-weekly, yearly).
- **Linear gradient (G):** A constant amount by which each consecutive cash flow increases (or decreases) in a linear gradient series.

- **Linear gradient series:** A series of cash flows in which each consecutive cash flow is larger or smaller than the previous by a constant dollar amount.
- **Nominal value:** The numerical value of a piece of currency (e.g. \$20.00 for a twenty-dollar bill).
- **Period (N):** Length of time a series of cash flows covers.
- **Present value (P):** The nominal value of a cash flow at a set “present” date. The present date can be specified as needed.
- **Principal:** The amount of money lent/borrowed.
- **Purchasing power:** A measure of the quantity of goods and services that can be purchased with a unit of currency.
- **Time value of money (TVM):** The concept of a dollar today being worth more than a dollar tomorrow.
- **Uniform series:** is a cash flow series in which the same amount of money (**annuity**) is paid or received in two or more sequential periods. Also called an **annuity** or **equal-payment series**.

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## 3.7 Supplementary Resources

### Supplementary Resources

- See [inflationcalculator.ca](http://inflationcalculator.ca) or [statcan.gc.ca](http://statcan.gc.ca) for more information on the CPI, purchasing power of money and relevant calculations, as well as the basket weights.

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# Chapter 4: Financing

## Introduction

Businesses often face decisions such as buying large pieces of equipment or embarking on expensive construction projects. These types of endeavours often require the company to spend significant amounts of money that the company might not have readily available. As a result, they often have to raise funds in some way. This may involve activities such as selling capital assets, issuing shares or, perhaps most commonly, borrowing from sources such as financial institutions or private investors. In this chapter, we are going to focus on the latter. Such borrowing arrangements are generally referred to as **loans**, as the **lender** loans money to the **borrower**.

In everyday life, perhaps most frequently encountered forms of borrowing include car loans, credit cards and house mortgages. These involve spending a money you don't have to obtain a product now and eventually repaying the borrowed money plus interest. Typically, repayment is accomplished through periodic payments, e.g. monthly, or as a lump sum payment. These concepts apply not only to personal spending but also to business expenditures. In all cases, borrowing money is not a decision to be taken lightly. One must understand the advantages and disadvantages of borrowing money and the consequences of debt.

So, what are the benefits and implications of borrowing money? The benefit to an individual or a company is that they can undertake projects or purchases without having to save all of the money in advance. For an individual, this might mean they can buy a car now rather than saving for several years before buying it. For businesses, this can have even more significant implications: it can allow them to buy equipment to increase production and, therefore, allow it to increase revenues. In other sectors, this may enable them to undertake capital projects such as new bridges, buildings, factories or plants, which may generate revenues for many years to come. As we have seen in Chapter 3, a dollar today is worth more than a dollar tomorrow, so companies want to start generating revenues today.

However, one must also realize that there are consequences to spending money before you actually have it. That is, there are costs associated with borrowing money. The most apparent cost is the interest charged on the amount borrowed. Paying interest essentially increases the cost of the equipment or project. Also, you run the risk of not being able to repay the loan, in which case the lender might repossess the company's assets or may just lose money. Thus, debt exposes both the borrower and the lender to more risk. Therefore, the more debt a company has, the less they're able to borrow until some of it is paid off.

The goal of this chapter is to introduce and discuss some of the debt and financing concepts commonly dealt with by individuals and businesses. This chapter will delve further into interest and compounding concepts covered in chapter 3, and enable students to analyze more complex situations.

### Key Concepts

- Benefits and costs of borrowing
- Loans, mortgages, and leases
- Compounding frequency and payment frequency
- Nominal vs. effective annual interest
- Loan payments
- Annuities and perpetuities

### Learning Objectives

After completing this chapter, students should be able to:

- Explain motivations for, and implications of, borrowing
- Compare financing options such as leases, loans, and cash purchases
- Calculate effective interest rates based on the nominal interest rate, payment frequency, and compounding frequency
- Calculate payment amounts, interest paid, and remaining balances for loans and mortgages
- Develop amortization schedules
- Evaluate annuities and perpetuities

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## 4.1 Loans

Loans can have many conditions associated with them, which can vary from loan to loan. For example, some may be simple such as a one month loan from a friend, some are more complex, such as a long-term loan from a financial institution to build a factory. Because loans can be so diverse, categories have been developed to help communicate some of the conditions of the loan. We will first introduce the most common conditions of loans and then explain how they are categorized based on these conditions.

### 4.1.1 Conditions of the loan

There are several details (i.e. conditions) that may vary or be negotiated between the borrower and the lender. These are generally agreed to and defined in a contract before money changes hands. The most common conditions include the:

- **Principal:** The amount of money lent/borrowed.
- **Amortization period:** The total length of time over which the borrower is expected to repay the loan.
- **Interest rate:** The cost of borrowing money expressed as a percent over a time period. (Refer to Chapter 3 Section 3.2 for more detail). When the interest rate is stated annually, it is called an **annual percentage rate (APR) or nominal interest rate**.
- **Compounding frequency:** This defines how often interest is calculated for a loan. It is usually expressed as the number of compounding periods per year.
- **Payment frequency:** This defines how often payments are made. E.g. weekly, monthly, annually.
- **Term of the loan:** The length of time the loan contract is in effect. This can be shorter than the amortization period. In such cases, a new loan contract is negotiated at the end of the term, unless the remaining principal is repaid.
- **Collateral:** This is an asset that belongs to the borrower that the lender can take if the borrower does not repay the loan. Thus, we say collateral is pledged as security on the loan.

All of these conditions are specified in the loan contract. For example, a company decides to borrow \$300,000 to purchase new equipment. The loan contract states 4% APR for 5 years, compounded semi-annually with monthly



payments. So, the principal is to be repaid over a 5-year period. To get the loan the company pledged a storage facility they own as security.

Here, the principal is \$300,000. The amortization period is 5 years. The interest rate is 4%. The term of the loan is 5 years. The compounding frequency is semi-annual, or 2 times per year. The payment frequency is monthly, or 12 times per year. Storage facility is used as collateral.

Some of these conditions determine the size of the loan payments and how much interest will need to be paid over the term of the loan, therefore affecting the total amount of money the borrower must repay. This is illustrated in Figure 4.1 below

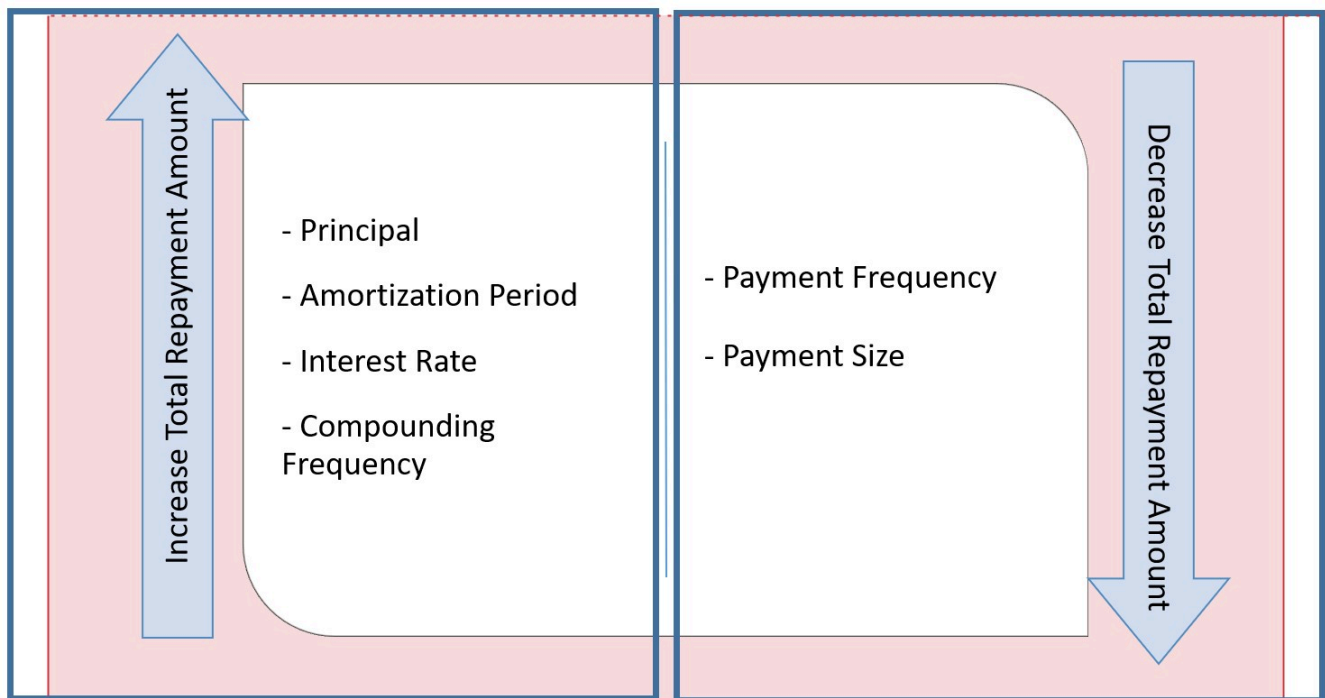


Figure 4.1. The Relationship Between Some of the Loan Conditions and Total Repayment Amount

As noted in the Figure 4.1, **principal**, **amortization period**, **interest rate** and **compounding frequency** positively correlate with the total amount of money the borrower must repay. The larger the **principal**, the more interest the borrower must pay, generally through higher loan payments. Longer **amortization periods** allow more time for interest to be applied to the balance of the loan, meaning that total interest increases with term length. The higher the **nominal interest rate**, the larger the interest amount will be accumulated throughout the term of the loan. Increasing **compounding frequency** increases the total amount of interest to be paid. (We will discuss the mechanism behind the effect of compounding frequency on the loan in section 4.2.1.) **Payment frequency** and **payment size** are negatively correlated with the total repayment amount. The more often payments are made and the bigger they are, the faster the loan is paid off, and the less interest is accumulated on the loan.

#### 4.1.2 Type of Loan

Types of loans are basic loan categories that have similar conditions. Knowing and understanding the implications of different types of loans is important in analyzing possible financial scenarios. It is important to be able to distinguish between them and be able to identify their main attributes.

This section will discuss the most common types of loans:

- Amortized loans
  - Conventional loans
  - High-rate loans
  - Collateral loans
  - Fixed-rate loans
  - Variable-rate loans
  - Open loans
  - Closed loans
- Interest-only loans

We will also discuss *leases* because they are a common alternative to loans. For example business owners may choose to lease office space rather than buying an office building.

All of these options can have their own benefits and drawbacks depending on the situation. By understanding how these types differ in terms of loan conditions, we can assess these advantages and disadvantages.

### 4.1.2.1 Amortized Loans

Amortized loans are the most common type of loans in Canada. In **amortized loans** the borrower repays the lender by making repeated, typically equal, payments at regular intervals, e.g. monthly, until the loan is repaid. Thus, this is called an amortized loan, because a total period of time for loan repayment (amortization period) was agreed to in advance, and affects other conditions of the loan, including the size and frequency of the loan payments. Each payment is uniform and includes some money to pay interest and some to repay the principal. Payments are made until the loan is repaid in full. Amortized loan calculations will be covered in Section 4.2.

Often, **down payments** are required by the lender. A down payment is the amount that the borrower must pay upfront to secure the loan or mortgage. For example, to get a \$400 000 loan, the borrower might have to pay a 10% down payment, which would be \$40,000 in this case (10% of the \$400 000 principal). Down payments reduce the risk to the lender.

Amortized loans are often described based on three different characteristics: 1) the amount of the down payment, 2) the interest rate and 3) repayment restrictions. For example, you may hear someone refer to a conventional fixed-rate closed loan. So, what does this mean? We will explain this terminology (as well as the alternatives) below.

First, a loan can be conventional, high-rate or collateral. These types reflect the amount of risk for the lender. To reduce their risk, lenders may require collateral. Often, the collateral is the asset being acquired through the loan (e.g. what the borrower is trying to buy).

For example, say a business decides to purchase an office building, that is worth \$1 million. If they borrow \$1 million and use the building as collateral, the lender can take the building. Suppose the company is unable to repay the loan. If the bank sells the collateral for \$1 million, they can get all their money back. However, they may not be able to sell it for \$1 million. If they get less than \$1 million, say \$900,000, they will lose money on the loan. If the lender tells the borrower that they need to give them a 10% down payment, that is \$100,000, they would get all of their money back even if they were able to only sell the building for \$900,000. Thus, requiring a higher down payment reduces the lender's risk of losing money.

Instead of a down payment, let's say the lender required that the borrower used a more expansive building that they already own (say \$1.5 million) as collateral. Now if the lender doesn't repay the \$1 million loan, the lender can take the more expansive building and sell it to get their money back. Even if they can't sell it for the initial value of \$1.5 million or they sell it at a discount rate to sell it faster (say they sold it for \$1.3 million), they still get all their money back.

These examples comprise the three categories below:

- **Conventional loans:** When the loan amount is less than or equal to 80% of the value of the asset. Typically, this is achieved by making a down payment of at least 20% of the asset's value (purchase price).
- **High-rate loans:** When the amount borrowed is greater than 80% of the value of the asset. In these cases, the interest rate is usually higher due to the increased risk to the lender, as discussed above.
- **Collateral loans:** When another asset (not the one being purchased) is used as a collateral in the loan agreement.

Depending on *the interest rate* loans can be:

- **Fixed-rate loans:** The interest rate remains constant throughout the term of the loan.
- **Variable-rate loans:** The interest rate can fluctuate over time. It is often set in relation to a "prime rate". In Canada, this is often the Bank of Canada's prime rate, which is set by the government and reflects the lending rate between banks.

Additionally, loans differ depending on *the repayment restrictions*, that is the ability to make payments in addition to the agreed repayment schedule.

- **Open loans:** Allow the borrower to make additional payments on the principal without penalty. This enables the borrower to finish off paying off the loan early and thereby reducing the total amount of interest paid.
- **Closed loans:** Charge the borrower a penalty if the borrower decides to make any additional payments

towards the loan during the term. Historically, in Canada any extra payments were penalised, however in the recent years, these loans have permitted more flexibility, although the amount and type of additional payments may still be restricted. (Why do banks penalize for repaying the debt early?)

Since, as mentioned above, amortized loan payments include some money to pay interest and some to repay the principal. As the borrower makes payments they reduce the amount that they owe, thus they typically build **equity** (refer to Chapter 1). Essentially, this is how much of the asset purchased through an amortized loan the borrower has paid off and owns now. Think of it this way, if you buy a house with a 10% down payment, you “own” 10% of the house, while the lender “owns” the other 90%. As you make payments towards the loan, the proportion of the house that you “own” increases.

## Mortgage

A special case of an amortized loan is a **mortgage**, which is typically used to buy houses, land and/or buildings. For example, many people apply for mortgages to buy houses. Companies can get mortgages to purchase new facilities to increase production. The difference between mortgages and other amortize loans is that many other loans are unsecured, in contrast, mortgages require the property that is being purchased to be used as collateral, i.e. the owner cannot sell the property without fully repaying the mortgage. Another difference is that for many mortgages the term is substantially shorter than the amortization period, which means a loan agreement is renegotiated multiple times before the loan is fully repaid.

Some features of mortgages are the same as those of amortized loans. Like any amortized loan, mortgages allow the borrower (mortgagee) to buy expensive assets earlier (i.e. without having to save up the full purchase price in advance). And, as with other amortized loans, mortgages generally involve borrowing a lump sum of money upfront and paying it back with regular payments.

A summary of the key differences includes:

- The term is typically shorter than the amortization period
- Mortgages are for real estate
- Properties are typically held as collateral
- There are specific laws surrounding mortgage agreements in Canada
- Interest is typically compounded on a semi-annual basis
- The maximum amortization period for housing
- Down payment requirements, insurance for housing

### 4.1.2.2 Interest-only Loans

Although uncommon in Canada, another type of loan offered is the **interest-only loan**. Like amortized loans, interest-only loans involve regular, equal payments made to the lender. In an interest-only loan, however,

payments do not affect the principal at all. All payments made are only paying interest on the loan. When the term of the loan ends, the loan will have to be either paid off, or a new loan obtained. Interest-only loans typically require collateral.

An advantage of interest-only loans is that during the term the borrower makes smaller payments, since none of the principal is being repaid. Small payments may be beneficial to borrowers in some scenarios, such as when the income or revenue is currently low but certain to increase in the future. For example, if a company is expanding its production to meet rising demand, and needs money to increase production, an interest-only loan would allow them to keep payments low in the short-term, so they can dedicate as much money as possible towards production. As their sales (and ideally profits) are realized from the increased production capacity, the company will be in a better position to repay the principal.

One drawback of interest-only loans is that the borrower pays more interest for 2 reasons. 1) Typically, interest only loans have higher interest rates than the amortized loans. 2) Interest is calculated on the principal amount for the entire duration of the loan. Another drawback is that since the payments are made only towards interest, the borrower is not building equity on the asset.

### 4.1.2.3 Leases

Instead of getting a loan to finance purchase, another option may be to lease the item. The concept of a leasing is very similar to renting, except that a leasing agreement is usually for longer periods of time. For example, instead of a month-to-month rental agreement, a lease may last several years. Owners take into account several factors when determining how much they will charge the user(lessee) to lease the item. These can include the initial purchase price of the item, ongoing costs (e.g. maintenance, utilities), and the anticipated resale value.

For the most part, you can think of leasing as long-term renting. Periodic payments are made in exchange for being able to use the asset without buying it. After the lease agreement expires, the asset or equipment is returned to the owner and no more payments are made. Some lease agreements, however, give the lessee the option of buying the asset after the lease expires (typically for a predetermined amount). Car leases, for example, often contain the option of purchasing the vehicle after the leasing agreement is over.

For the lessee, some of the key advantages of leases are:

- Low initial cost compared to loans – no down payment, usually just the first lease payment and administrative fees.
- Known, fixed payments for the entire period of the lease.
- Fewer ownership responsibilities or costs (e.g. repairs and property taxes).
- Avoid the burden of selling (or disposing of) the asset when it is no longer needed.
- Leases allow you to “try before you buy”, so you can make sure the lease item will suit your need before making a substantial investment to purchase the item.

Some of the key disadvantages of leases for the lessee are:

- Restrictions on use and ability to make modifications. E.g. number of hours or kilometers of use, extent of renovations.
- Lessee does not build equity on the asset.
- Potentially more expensive long term.

There are situations where leasing is the best option. In particular, leases may be beneficial to the lessee when the equipment is only needed for a short period of time, or the lessee does not have the expertise in using or maintaining specialized equipment.

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## 4.2 Loan Calculations

When acquiring a loan, we need to determine the loan payments and their structure. In order to do so, there is one key concept that must be understood: **effective interest rates**.

### 4.2.1 Nominal and Effective Interest Rates

When compounding frequency and payment frequency do not match, the nominal interest rate does not accurately reflect the overall interest actually charged (or received). For example, imagine you have \$1000 on your credit card, 12% APR compounded monthly and you want to repay the amount owed in a lump sum payment in 1 year. You might think that you will be charged \$120 in interest. But in fact, the actual interest amount is \$126.83! Why is that? We will go through the details in example 4.1, but this illustrates that the nominal interest rate does not tell the whole story, which brings us to effective interest rate...

The 12% APR, compounded monthly means that interest is accumulated at 1% per month, i.e. 12%/12 months. If you wanted to calculate the actual interest in 1 year, we use the future value formula  $F=P(1+i)^N$ . This is how you find the \$126.83. So, on a loan you would end up paying more than 12% interest. This makes the APR awfully misleading, doesn't it? By paying \$126.83, you can see that you are effectively paying 12.68%. This is the effective interest rate.

The effective interest rate is used to find the actual interest accumulated on the loan taking into account the compounding and payment frequencies. Since introducing compound interest in Chapter 3, we have purposefully avoided this issue until now by using equal time periods for the interest rate and compounding and payment frequencies. For example, we have said 10% per year, compounded annually with annual payments. However, in practice the interest period, compounding and payment frequencies rarely align. Frequently, a loan contract lists an annual interest rate, but the interest compounds more than once per year, and the payment frequency can also differ. For example, if you look at car advertisements, you may see the interest rate is listed as an APR. But look at the fine print! It may say that interest is compounded daily or monthly, and payments must be made monthly or bi-weekly. This may have substantial effect on the total amount of interest paid. Suddenly the car costs a lot more than you expected. Clearly, the effective interest rate is an important concept.

#### 4.2.1.1 Interest rate per compounding period

Let's first take a look at how the compounding frequency affects the actual interest accumulated on the loan. As

mentioned in section 4.1.1, compounding more frequently will increase the total interest accumulated. This may be best shown by considering the following interest rate discussion related to interest calculations for a credit card.

Suppose the terms of a credit card state “12% APR compounded monthly.” This tells us:

- Nominal interest rate, or APR = 12%
- Interest is compounded at the end of each month (12 times a year)

This does NOT mean that the account earns 12% interest every month. It means that a portion of the 12% is applied to the credit card balance each month. To find this portion, we calculate the interest rate per compounding period by dividing the nominal rate (APR) by the number of compounding periods in a year:

$$i = \frac{r}{M}$$

where

$i$  = interest rate per compounding period,

$r$  = nominal interest rate (APR), and

$M$  = number of compounding periods per year.

In our case, the interest rate per compounding period is:

$$i = \frac{12\%}{12} = 1\%$$

This means that when the terms state “12% APR compounded monthly”, 1% interest is applied to the account at the end of each month.

Now that we know the interest rate that is applied each compounding period, we can calculate the value of the account and the total interest accumulated after 1 year. For this we use the future-value formula (equation 3.6):

This formula requires the interest rate and the compounding frequency to have the same time basis. So, we use the interest rate per compounding period that we have just calculated. So, if you had \$1000.00 on the credit card with 1% interest rate per month, the value of the account after 1 year would be:

The total interest accumulated is:

$$F - P = \$1126.83 - \$1000.00 = \$126.83.$$

Table 4.1 illustrates how interest is accumulated in this scenario over the first year.



Month	Principal	Interest
January	1000.00	10.00
February	1010.00	10.10
March	1020.10	10.20
April	1030.30	10.30
May	1040.60	10.41
June	1051.01	10.51
July	1062.52	10.62
August	1072.14	10.72
September	1082.86	10.83
October	1093.69	10.94
November	1104.62	11.05
December	1115.67	11.16
<b>TOTAL</b>	<b>1126.83</b>	<b>126.83</b>

Now what would the total interest be after 1 year if we were to calculate interest with 12% APR assuming annual compounding? The future-value would be:

The total interest paid in the second scenario would thus be:

$$F - P = \$1120.00 - \$1000.00 = \$120.00.$$

Let's compare these scenarios. In both cases, the APR is 12%. When interest is compounded annually, the \$1000 account earns \$120 interest after one year. When 12% APR is compounded monthly, the total interest paid is higher (\$126.83). Why is that? As shown in Table 4.1, in the first month, the interest earned is \$10, or 1% of the principal. This brings the account balance to \$1010. In the second month, the interest is 1% of the \$1010 balance, which equals \$10.10 in interest... and so on. This means that early interest accumulates its own interest, which then goes on to accumulate its own interest, and so on. Thus, the compounding frequency changes the amount of interest that accumulates in an account, even if the APR is unchanged: by compounding more frequently interest accumulates more often, which enables more interest to accumulate its own interest. **If the APR is kept constant, increasing the compounding frequency increases the amount of interest earned over the same time period.**

## Interest Rate per Compounding Period Example

Cameron won \$5000.00 in a lottery and decided to put the money in a savings account. The bank offered Cameron 2 savings account options:

- A. An account that earns 3% (APR) compounded quarterly or
- B. An account that earns 2.5% (APR) compounded monthly

What account should Cameron choose if he wants to pick the one that earns the most interest?

### Solution

To find out which account is a better option for Cameron we need to calculate the interest each account earns after 1 year and compare them. Note that in this example it does not matter whether the interest is calculated after 1 year or multiple years, since the relation between the accounts will be the same whether we calculate the values after 1 year, 2 years or 20 years. Thus, calculating the interest after 1 year is sufficient.

### Option A

The first account earns 3% (APR) compounded quarterly. This means that the portion of 3% is applied to the account 4 times a year.

Before we can use the future-value formula 3.6, we need to bring the interest rate and the compounding frequency to the same time basis – quarterly. Using formula 4.1, we calculate the *interest rate per compounding period* for the savings account:

$$i = \frac{3\%}{4} = 0.75\%$$

So, 0.75% is applied to the savings account on a quarterly basis.

Next, we use the future-value formula 3.6 to find the value of the account after 1 year (4 compounding periods) with the interest rate per compounding period 0.75%:

$$F = \$5000.00(1 + 0.0075)^4 = \$5151.70$$

Thus, the interest the account accumulates after 1 year is:

$$F - P = \$5151.70 - \$5000.00 = \$151.70$$

### Option B

The second account earns 2.5% (APR) compounded monthly. This means that the portion of 1.5% is applied to the account 12 times a year.

To bring the two to the same time basis – monthly, we calculate the interest rate per compounding period for the account using formula 4.1:

$$i = \frac{2.5\%}{12} = 0.21\%$$

So, 0.21% interest is applied to the savings account monthly.

Next, we use the future-value to find the value of the savings account in option B after 1 year (12 compounding periods) with the interest rate per compounding period 0.21%:

$$F = \$5000.00(1 + 0.0021)^{12} = \$5127.47$$

Hence, after 1 year the account accumulates in interest:

$$F - P = \$5127.47 - \$5000.00 = \$127.47$$

Therefore, the savings account in option A earns more interest than the savings account in option B (\$151.70 vs \$127.47). Cameron should choose a savings account that pays 3% (APR) compounded quarterly.

### 4.2.1.2 Effective Annual Rate

In the previous example, interest was compounded monthly, so calculating the accumulated interest over the year was not an overly demanding task. However, what if the compounding frequency is very high?

#### Example 4.2

Suppose you decide to open an investment account. The terms of the account state 6.75% APR, compounded daily. If you invest \$10,000, how much would you have in a year?

We begin by calculating the interest rate per compounding period to bring the interest rate to the same time basis as the compounding frequency. Because interest is compounded daily, we divide the nominal interest rate by 365 days – number of compounding periods:

$$i = \frac{6.75\%}{365} = 0.0185\%$$

To find the interest accumulated over the year, we first find the future value of the investment account after 1 year using the future-value formula 3.6:

$$F = \$10,000.00(1 + 0.000185)^{365} = \$10,698.24$$

This means that the interest on the investment after a year is

$$F - P = \$10,698.24 - \$10,000.00 = \$698.24$$

Table 4.2 illustrates how interest is accumulated over the first year with 6.75% APR compounded daily.

Day	Principal	Interest
January 1	\$10,000.00	\$1.85
January 2	\$10,001.85	\$1.85
January 3	\$10,003.70	\$1.85
...	...	...
March 1	\$10,109.70	\$1.87
March 2	\$10,111.57	\$1.87
...	...	...
June 1	\$10,283.16	\$1.90
June 2	\$10,285.06	\$1.90
...	...	...
September 1	\$10,459.59	\$1.93
September 2	\$10,461.52	\$1.93
...	...	...
December 31	\$10,696.26	\$1.98
<b>January 1 (next year)</b>	<b>\$10,698.24</b>	

Table 4.2 Interest accumulation with 6.75% APR compounded daily

As shown in Table 4.2, with daily compounding the investment account accumulates 0.0185% interest on its balance every day, resulting in \$698.24 interest after 1 year.

Now how much interest would be accumulated after 1 year in this scenario if interest was compounded annually? The future-value of the account would be:

$$F = \$10,000.00(1 + 0.067)^1 = \$10,670.00$$

So, the total interest accumulated in this scenario would be:

$$F - P = \$10,670.00 - \$10,000.00 = \$670.00.$$

Similarly to example 4.1, the interest in the daily-compounding scenario is greater compared to the one where the interest is compounded annually due to a higher compounding frequency (\$698.24 vs \$670.00). To compare these two scenarios, we applied the approach introduced in example 4.1: we used the interest rate per compounding period and the number of compounding periods per year to get the value of the account after 1

year. Now, however, in the daily-compounding scenario, calculating interest in all 365 periods becomes an arduous task.

There is another way to solve this problem. Notice the interest amounts in both scenarios. In the annual-compounding scenario, the interest after 1 year is \$670.00, which is precisely 6.75% of the investment and is the annual nominal interest rate (APR). In the daily-compounding scenario, the interest after 1 year is \$698.24, which corresponds to a 6.982% interest rate. This adjusted annual interest rate is termed an **effective annual rate (EAR)**.

An **effective annual rate (EAR)** is a restatement of the nominal interest rate (APR) adjusted on the compounding frequency. The EAR is used as an annual interest rate to calculate how much interest on the account will be earned yearly if it is compounded at a frequency that is other than once per year. For example, when calculating the future value of the investment account using the future-value formula 3.6, we use the EAR instead of the nominal interest rate (APR) and number of years as  $N$ .

The EAR can be derived in the following way:

- 
1. We start by using the formula for finding the interest rate per compounding period 4.1. Recall that interest charged per compounding period is equal to the APR divided by the number of compounding periods in a year, or  $\frac{r}{M}$ .
- 

$$i_M = \frac{r}{M}$$

- 
2. Over the course of the year there are  $M$  compounding periods. To get the future value of the loan after 1 year, we use the future-value formula, substituting the interest rate per compounding period and the number of compounding periods in 1 year.
- 

$$F = P\left(1 + \frac{r}{M}\right)^M$$

- 
3. We can also define the value after one year,  $F$ , in terms of  $P$  and an annual interest rate,  $i_a$ . Since we use an adjusted interest rate which represents all the interest earned in one year, the future value of the loan after 1 year can be expressed as
- 

$$F = P(1 + i_a)^1$$

- 
4. We now have two equations for  $F$  – (2) and (3). After setting them equal to each other, we solve for  $i_a$ . The obtained formula is
- 

$$\begin{aligned} P(1 + i_a) &= P\left(1 + \frac{r}{M}\right)^M \\ 1 + i_a &= \left(1 + \frac{r}{M}\right)^M \\ i_a &= \left(1 + \frac{r}{M}\right)^M - 1 \end{aligned}$$

Thus, the formula for calculating the EAR is

$$i_a = \left(1 + \frac{r}{M}\right)^M - 1 \quad (4.2)$$

where

$i_a$  = the effective annual rate, or EAR,

$r$  = the nominal interest rate, or APR,

$M$  = the number of compounding periods per year.

Let's refer back to example 4.2. The investment made is \$10,000 at 6.75% APR, compounded daily. We can now use the EAR formula to convert APR to an annual interest rate, then use it in the future-value formula 3.6 to find the value of the account after one year.

First step is finding the EAR. The nominal interest rate,  $r = 6.75\% = 0.0675$ . Since the interest on the investment account is compounded daily, . Using formula 4.2, we obtain the EAR:

Now, we use the future-value formula 3.6 with 6.982% as the interest rate to find how much the account will be worth after 1 year:

Thus, the total interest accumulated after 1 year is:

$$F - P = \$10,698.20 - \$10,000.00 = \$698.20$$

Let's take a look at another example involving the EAR.

### Effective Annual Rate Example

*Adele is thinking of getting a new credit card for her impulsive shopping needs. She is considering two different cards. Their interest rates and compounding frequencies are as follows:*

**A.** 18.5% APR, compounded monthly

**B.** 20% APR, compounded quarterly

*Which credit card should Adele get based on their borrowing rates? Assume annual payment schedule.*

### Solution

Adele should get a credit card that has a lower borrowing rate. To determine which card it is, we calculate the EAR for both credit cards and compare them.

### Option A

The nominal interest rate on the first credit card is 18.5% (. Interest is compounded monthly ( ). Substituting these values in formula 4.2 we calculate the EAR on the first credit card:

$$i_a = \left(1 + \frac{r}{M}\right)^M - 1 = \left(1 + \frac{0.185}{12}\right)^{12} - 1 = 0.2015$$

So, the effective annual rate on the first credit card is 20.15%.

### Option B

The nominal interest rate on the second credit card is 20% ( ). Interest is compounded quarterly ( ). The EAR is thus:

$$i_a = \left(1 + \frac{r}{M}\right)^M - 1 = \left(1 + \frac{0.2}{4}\right)^4 - 1 = 0.2155$$

So, the effective annual rate on the second credit card is 21.55%.

Comparing the two credit cards we see, that option A has a cheaper rate. Adele should get the first credit card.

Also, note that in both cases EAR is higher than the nominal interest rate (APR) due to the compounding frequency. This is something to keep in mind when budgeting or considering getting a loan.

#### 4.2.1.3 Effective Interest Rate

The effective annual rate (EAR) is used in scenarios when interest is compounded on a basis other than annual and payments are made yearly. However, oftentimes, we are faced with loan terms that require payments to be made more frequently. For example, when getting a car loan, you will most likely have a schedule of monthly payments, rather than yearly. In scenarios like these, we calculate what is termed an **effective interest rate (EIR)**.

An effective interest rate (EIR) formula is an adjusted formula of the EAR that accounts for different payment schedules. Derivation of the formula is similar to that of the AER formula. The only difference is that to get a more general form we express the number of compounding periods in a year as:

$$M = CK \quad (4.3)$$

Where

M = the number of compounding periods,

C = the number of interest periods per payment period, and

K = the number of payments per year.

Thus, the EIR can be derived as follows:

1. We start by finding the interest rate per compounding period using formula 4.1. This time, we substitute  $M=CK$  into the equation

$$i_M = \frac{r}{M} = \frac{r}{CK}$$

1. Over the course of the year there are  $C$  interest periods. To get the future value of the loan after 1 payment period, we use the future-value formula 3.6 substituting the interest rate per compounding period, and the number of interest periods in 1 payment period,  $C$ .

$$F = P \left( 1 + \frac{r}{CK} \right)^C$$

2. Remember, we can also define the value after one payment period,  $F$ , in terms of  $P$  and an effective interest rate,  $i$ . So, the future value of the loan after 1 payment period can be expressed as

$$F = P(1 + i)$$

3. Setting equations (2) and (3) equal to each other, we solve for  $i$ . As a result, we get the formula for calculating the EIR

$$P(1 + i) = P \left( 1 + \frac{r}{CK} \right)^C$$

$$1 + i = \left( 1 + \frac{r}{CK} \right)^C$$

$$i = \left( 1 + \frac{r}{CK} \right)^C - 1$$

(4) Thus, the formula for calculating the EAR is

$$i = \left( 1 + \frac{r}{CK} \right)^C - 1$$

Thus, the formula for calculating the EAR is  $i = \left( 1 + \frac{r}{CK} \right)^C - 1$  (4.4)

where

$i$  = the effective interest rate, or EIR,

$r$  = the nominal interest rate, or APR,

$C$  = the number of interest periods per payment period, and

$K$  = the number of payments per year.

Given a nominal interest rate and compounding frequency, the EIR can be calculated for any payments schedule using formula 4.4. For example, for 9% APR, compounded monthly, the effective rates for *monthly*, *quarterly*, *semi-annual*, and *annual* payments are shown in Figure 4.2:



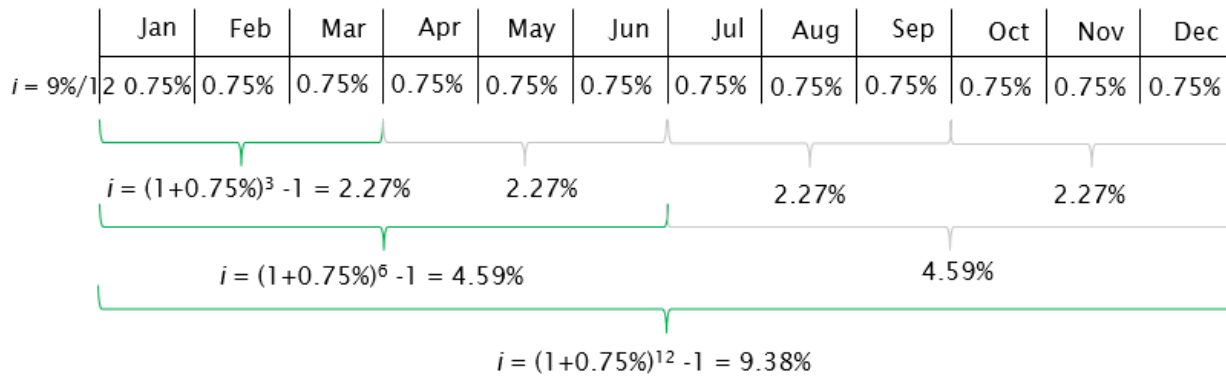


Figure 4.2 The Effective Interest Rate for Different Payment Frequencies

Note that the EAR is a partial case of the more general form, the EIR. When payments are made annually, from equation 4.3 we get  $K=1$  and, thus,  $M=C$ . Consequently, using the fact that  $M=C$ , we get the formula for EAR (4.2) from the formula for EIR (4.4). Similarly, the interest rate with compounding period is also a partial case of the EIR, when payment and compounding frequencies are the same.

These partial cases together with the two cases when compounding frequency is higher than the payments frequency and when compounding frequency is lower than the payment frequency are explored in more detail in the example below.

#### Effective Interest Rate Example: different compounding and payment frequencies scenarios

Irvin has 4 savings accounts from different banks. The terms and payment schedules for these accounts are as follows:

1. 8% APR, compounded monthly, payments are made monthly
2. 8% APR compounded monthly, payments are made quarterly
3. 8% APR compounded quarterly, payments are made monthly
4. 8% APR, compounded monthly, payments are made annually

What are the effective interest rates on each of these accounts?

#### Solution

This example represents **4 scenarios with the same nominal interest rate, but different compounding and payment frequencies.**

#### Account 1

In the first scenario, the interest is compounded monthly and payments are made monthly, so the **compounding frequency is equal to the payment frequency.** In this case, we use the interest rate per compounding period

formula (4.1): we divide the nominal interest rate by the number of interest periods, which is equal to 12, to get the effective monthly interest rate for scenario 1:

$$i_1 = \frac{8\%}{12} = 0.67\%$$

Another way to get the EIR is to use the general EIR formula (4.4). We do this calculation to check the answer. Substituting  $r = 8\% = 0.08$ ,  $C = 12$  and  $K = 12$ , we get:

$$i_1 = \left(1 + \frac{r}{CK}\right)^C - 1 = \left(1 + \frac{0.08}{12 \times 12}\right)^{12} - 1 = 0.00669$$

So, the effective monthly interest rate on the first savings account is 0.67%.

### Account 2

In the second scenario, the interest is compounded monthly and payments are made quarterly, so **the compounding frequency is higher than the payment frequency**. One payment period contains 3 interest periods. We use the general EIR formula (4.4) to calculate the effective interest rate per quarter. Substituting  $r = 8\% = 0.08$ ,  $C = 3$  and  $K = 4$ , we get

$$i_2 = \left(1 + \frac{r}{CK}\right)^C - 1 = \left(1 + \frac{0.08}{3 \times 4}\right)^3 - 1 = 0.0201$$

So, the effective interest rate per quarter on the second savings account is 2.01%.

### Account 3

In the third scenario, the interest is compounded quarterly and payments are made monthly, so **the compounding frequency is lower than the payment frequency**. One payment period contains  $\frac{1}{3}$  of an interest period. Like in the previous scenario, we use the general EIR formula (4.4) to calculate the effective interest rate per month. Substituting  $r = 8\% = 0.08$ ,  $C = \frac{1}{3}$  and  $K = 12$ , we get:

$$i_3 = \left(1 + \frac{r}{CK}\right)^C - 1 = \left(1 + \frac{0.08}{\frac{1}{3} \times 12}\right)^{\frac{1}{3}} - 1 = 0.00662$$

So, the effective interest rate per month on the third savings account is 0.66%.

### Account 4

In the fourth scenario, interest is compounded monthly and payments are made annually, so **payment are made once a year**. Recall that this is a partial case when we calculate the effective annual rate (EAR) using formula 4.2 and  $M = C = 12$ . The EAR is:

$$i_a = \left(1 + \frac{r}{M}\right)^M - 1 = \left(1 + \frac{0.08}{12}\right)^{12} - 1 = 0.0830$$

So, the effective annual rate on the fourth savings account is 8.30%.

Summarized formulas for the 4 scenarios depending on compounding and payment frequency can be found in Table 4.3 below.

Scenario				
1. The Compounding frequency is equal to the Payment frequency (or there are no payments)	$C = 1$	Interest rate per compounding period formula	$i = \frac{r}{M}$	(4.3)
2. The compounding frequency is higher than the Payment frequency	$C > 1$	Effective interest rate formula	$i_2 = \left(1 + \frac{r}{M}\right)^{\frac{M}{K}} - 1$	(4.4)
3. The compounding frequency is lower than the Payment frequency	$C < 1$	Effective interest rate formula	$i_3 = \left(1 + \frac{r}{M}\right)^{\frac{M}{K}} - 1$	(4.4)
4. Payment are made annually	$K=1$	Effective annual rate formula	$i_a = \left(1 + \frac{r}{M}\right)^M - 1$	(4.2)

Table 4.3 Effective interest formulas for corresponding compounding-payment frequency scenarios

Note that the Table 4.3 contains suggested formulas. In all four scenarios, the general EIR formula (4.4) can be used. The results will be the same. Which formula to use is just a matter of preference.

#### 4.2.1.4 Changing Interest Rates

Previously in Sections 4.2.1.1 through 4.2.1.4 we assumed the same interest rate throughout the term of the loan. However, while the interest rate is fixed for many short-term loans, many long-term loans allow the interest rate to be adjusted. As discussed in Chapter 3, the interest rate is set to account for inflation and risks associated with borrowing money. In contrast to short-term loans, it is a lot harder to project inflation and risks over longer periods of time to guarantee a fixed interest rate. The economic situation in the region is subject to constant change: for example, when the inflation rate prognosis changes in 5 years due to government policy, the interest rates have to be adjusted accordingly. Thus, the ability to adjust interest rates is an important leverage to mitigate possible shocks and economic changes in the long run, and ensure the costs associated with borrowing money are met.

How do we approach loan calculations when the interest rate changes throughout the term of the loan? Let's take a look at an example that addresses this scenario.

##### Example 4.3

Suppose a company decides to take a \$20,000 loan for 5 years. The terms of the loan state 7% APR compounded monthly for the first 2 years and 10% APR compounded monthly for the last 3 years. Payments are made monthly. What is the total amount the company will have to pay in 5 years?

Using the cash flow diagram, this problem can be represented in the following way:

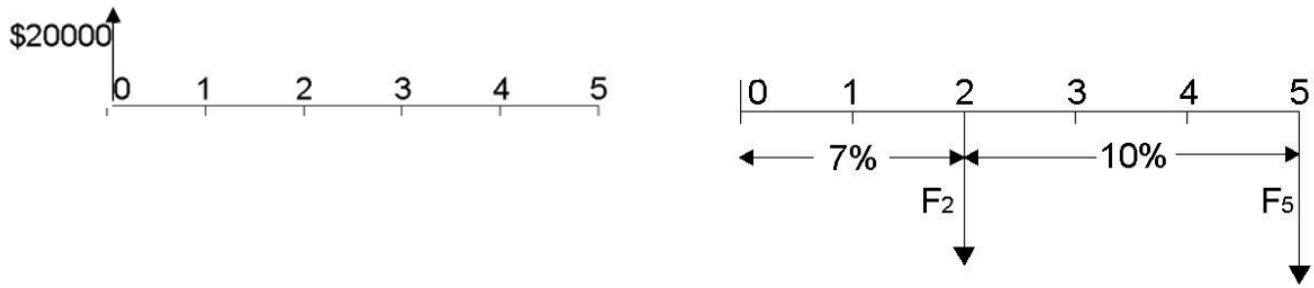


Figure 4.3 The Cash Flow Diagram for Example 4.3

One way to approach problems that involve changing interest rates is **to move the cash flows through each interest rate phase**. We have two interest rate phases in this example: the 7% and 10% phases, which we will call Phase 1 and Phase 2 accordingly.

### Phase 1

The example provides an annual percentage rate, however compounding and payment frequency are not annual. Thus, we need to adjust the interest rate to account for different compounding and payment frequency by calculating the effective interest rate.

To decide which formula to use for calculating the EIR, we look at the relationship between compounding and payment frequencies. In our scenario, the compounding frequency is equal to the payment frequency – monthly, so we calculate a partial EIR case – interest rate per compounding period, using formula 4.3:

$$i_{phase1} = \frac{r}{M} = \frac{7\%}{12} = 0.583\%$$

The next step is to calculate the future value of the loan at year 2. Interest is compounded 12 times a year, so in 2 years it is compounded 24 times. Using the future-value formula 3.6 we get:

$$F_2 = P_0 (1 + i)^N = \$20,000 (1 + 0.00583)^{24} = 22,994.29$$

So, the future value of the loan at the end of phase 1 is \$22,994.29

### Phase 2

Using the same approach as in phase 1, we first calculate the interest rate per compounding period for phase 2:

$$i_{phase2} = \frac{r}{M} = \frac{10\%}{12} = 0.833\%$$

Now we move the cash flow from year 2 to year 5 using the future-value formula 3.6. Notice that this time we use the future value of the loan at year 2 instead of present value of the loan. This is what we mean by moving the cash flow through each interest rate phase.

$$F_5 = F_2 (1 + i)^N = \$22,994.29 (1 + 0.00833)^{36} = \$30,996.80$$

Thus, the total amount the company will have to pay in 5 years is \$30,996.80.

### Changing Interest Rates Example

What is the total present value of the payments shown in Figure 4.4? The following interest rates are all compounded annually.

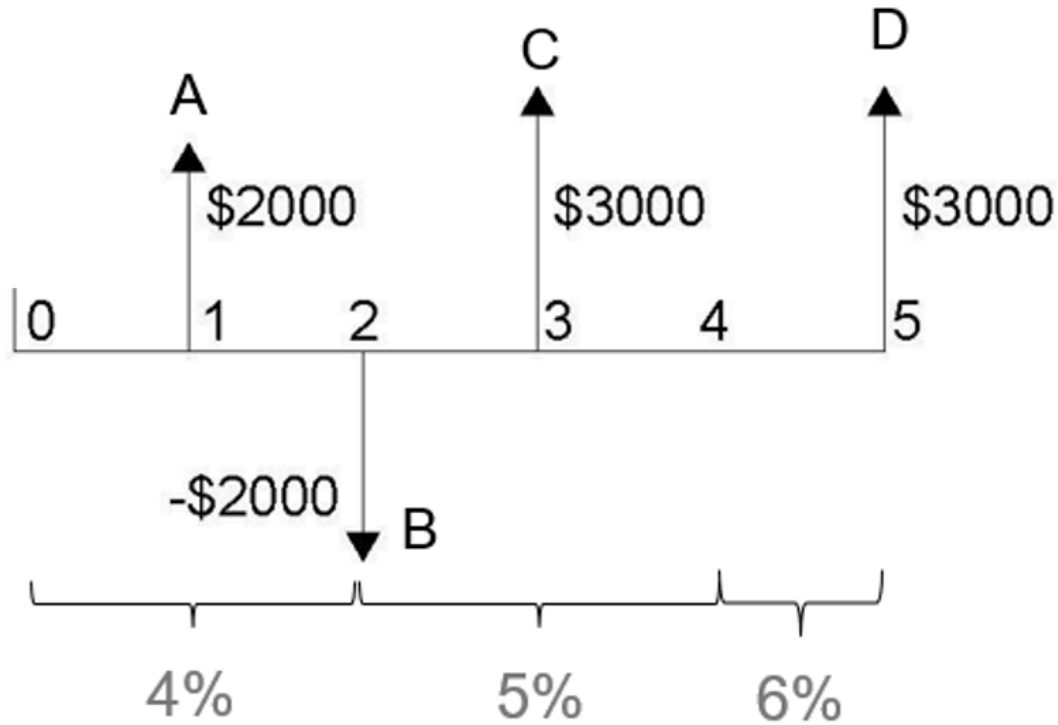


Figure 4.4 Payments schedule

### Solution

Step 1: To solve this problem, firstly, we have to move all payments individually through the interest periods. There are 4 payments A-D in the example. Because interest is compounded annually and payments are made annually, we do not have to adjust the interest rate.

### Payment A

The first payment is made in year 1. We use the present-value formula 3.8 introduced in Chapter 3 to calculate the value of payment A in year 0:

$$P_A = \frac{F_A}{(1+i_A)^{N_A}} = \frac{\$2000}{(1+0.04)^1} = \$1923.08$$

### Payment B

In the second year, we have a negative cash flow. Movement of the payment to year 0 does not require changing interest rate, so we use the present-value formula 3.8 to obtain the present value of the cash flow:

$$P_B = \frac{F_B}{(1+i_B)^{N_B}} = \frac{-\$2000}{(1+0.04)^2} = -\$1849.11$$

### Payment C

To discount the value of the third payment in year 3, we now have to account for the changing interest rates. Thus, we first discount the payment using 5% interest rate to find the value of the payment in year 2, then discount the payment using 4% interest rate to find its value in year 0:

$$F_{C2} = \frac{F_{C3}}{(1+i_{C3})^{N_{C3}}} = \frac{\$3000}{(1+0.05)^1} = \$2857.14$$

$$\text{and } P_C = \frac{F_{C2}}{(1+i_{C2})^{N_{C2}}} = \frac{\$2857.14}{(1+0.04)^2} = \$2641.59$$

You can also do this in one step:

$$P_C = F_{C3} \left( \frac{1}{(1+i_{C3})^{N_{C3}}} \frac{1}{(1+i_{C2})^{N_{C2}}} \right) = \$3000 \left( \frac{1}{(1+0.05)^1} \frac{1}{(1+0.04)^2} \right) = \$2641.59$$

### Payment D

Payment D is made in year 5. To discount this payment, we have to account for the interest rate changing 2 times: in year 4 and in year 3. Firstly, we discount the payment using 6% interest rate to find the value of the payment in year 4. Then we use 5% interest rate to move the payment to year 2. Finally, we use 4% interest rate to obtain a present value in year 0 of the payment. This could be done in multiple steps or in one step as follows.

Multiple-step:

$$F_{D4} = \frac{F_{D5}}{(1+i_{D5})^{N_{D5}}} = \frac{\$3000}{(1+0.06)^1} = \$2830.19$$

$$F_{D2} = \frac{F_{D4}}{(1+i_{D4})^{N_{D4}}} = \frac{\$2830.19}{(1+0.05)^1} = \$2567.07$$

$$D_D = \frac{F_{D2}}{(1+i_{D2})^{N_{D2}}} = \frac{\$2567.07}{(1+0.04)^1} = \$2373.40$$

One step:

$$P_D = F_{D5} \left( \frac{1}{(1+i_{D5})^{N_{D5}}} \frac{1}{(1+i_{D4})^{N_{D4}}} \frac{1}{(1+i_{D2})^{N_{D2}}} \right)$$

$$P_D = \$3000 \left( \frac{1}{(1+0.06)^1} \frac{1}{(1+0.05)^2} \frac{1}{(1+0.04)^2} \right)$$

Step 2: Now that we know present values of all payments, we sum them to find the total present value of all payments:

$$P_{total} = P_A + P_B + P_C + P_D = \$1923.08 - \$1849.11 + \$2641.59 + \$2373.40 = \$5088.96$$

So, the total present value of payments shown in Figure 4.4 is \$5088.96.

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# Chapter 5: Evaluation Tools

## Introduction

Suppose that in ten years you are the engineering manager at a large agricultural equipment manufacturing plant. You are faced with some important management decisions. For example, say there is a new piece of equipment available that would allow the company to manufacture plows more efficiently. The equipment is expensive but would shorten the time required to manufacture each plow. This could increase the number of plows made each year, and reduce the cost of each plow. The company has estimated the amount of production time the equipment would save on each plow, but they cannot be sure about how the demand for their plows will change in the coming years. Should the company purchase the equipment?

Even in this small example, there are many factors to consider when choosing whether to purchase the equipment. Some questions the company might need consider include:

- How long will the equipment last?
- How many more plows would need to be sold each year to offset the cost of the equipment?
- How accurate are their projections for future sales?
- Is the company likely to lose money on this purchase?
- How accurate do their production time estimates need to be to give them confidence in their decision?

In this chapter, we will discuss the primary analysis techniques that can be applied to this situation and many others like it. First, we will discuss the criteria and metrics used for decision making, and how to determine their thresholds. Then, we will cover several commonly used evaluation methods. This will include a discussion into how these methods can be applied to projects with different time scales. Finally, we will examine how different evaluation methods can be used to assess the riskiness of a project or investment.

## Key Concepts and Terms

- Minimum Acceptable Rate of Return (MARR) selection, cost of capital
- Net Present Value (NPV), Annual Equivalent Value (AEV), payback period, Internal Rate of Return



(IRR)

- Analysis period and project lives
- Risk assessment and evaluation

## Learning Objectives

After completing this chapter, students should be able to:

- Define and apply MARR and cost of capital
- Choose and apply the appropriate evaluation methods (payback period, NPV, AEV, etc.) for given problems
- Compare scenarios for different project or investment durations
- Evaluate risk by considering variability in estimates

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## 5.1 Rate of Return

Companies typically invest in projects to generate income. The rate of return is the profit from an investment divided by the initial investment, typically expressed as a percentage. For example, if you buy a \$1.00 lottery ticket and win \$10.00 (yielding a profit of \$9.00), then your rate of return would be  $\$9.00/\$1.00 = 900\%$ .

Often, people making investment decisions require a minimum rate of return to justify making the investment. Formally, this threshold or benchmark is called the MARR (minimum acceptable rate of return). Typically, a MARR would only be set by a business; however, an individual could also effectively have a MARR for their personal investments. For example, if you have a savings account that generates 1.5% interest, you might say that your personal MARR is 1.5%, because you would not invest in any projects that would earn you less than keeping your money in that account.

In a given situation, the MARR is chosen based on several factors:

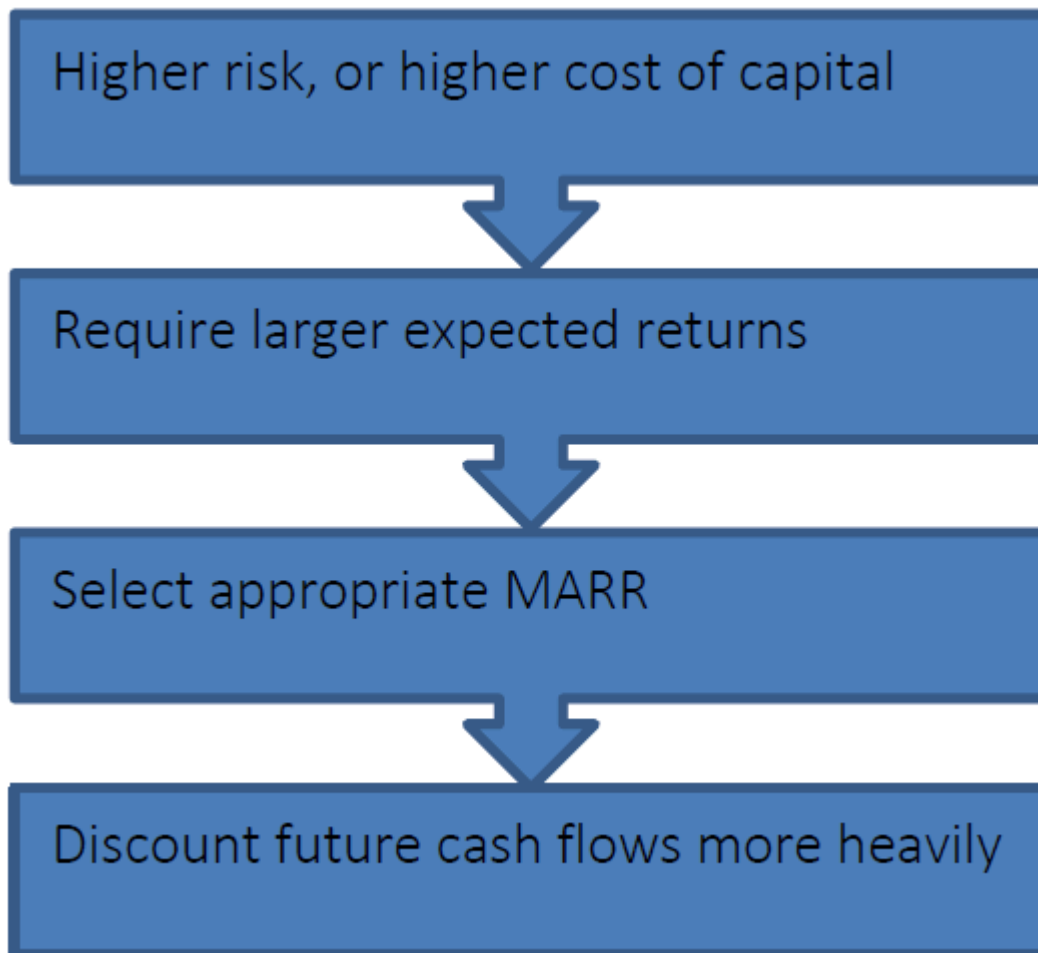
- **Opportunity cost.** As discussed in Chapter 3, an opportunity cost is the price of spending your money on one project or investment rather than another. When your money is “tied up” in an investment, you may miss out on investment opportunities that could have earned you more money.
- **Inflation.** The basics of inflation are also discussed in Chapter 3. Since most economies continuously experience inflation, the rising level of prices must always be considered when investing. The rate of return on an investment must be at least equal to the inflation rate to maintain purchasing power. If it is less than inflation, an investment may increase in dollar value while decreasing in actual purchasing power.
- **Cost of capital.** Getting the money you need to pay for a project often incurs costs. If the project is financed through debt (i.e. you borrow the money), then interest payments on that loan must be considered. Similarly, if the project is financed by investors (i.e. partners who have pitched in some money), these investors will expect a certain return on their investment. Thus, both of these financing methods, or ways of gaining capital, will cost you money, which is termed the “cost of capital”. Methods for determining the cost of capital for a project will not be covered in this text as they can be quite complex, extending beyond just interest, and returns to investors.
- **Risk.** The future cash flows of a project are almost never guaranteed or certain. Unforeseen circumstances can alter the amount or timing of expected cash flows; for example, bad weather might

delay construction on a project and cause unexpected costs, or a borrower might not repay a loan. The chosen MARR acts as a type of safeguard, ensuring riskier projects are only accepted if they have a higher expected return on investment. For example, investing your money in a small start-up company is generally riskier than investing it in a large, well established corporation because the start-up may be more likely to go out of business; however, the start-up also has the potential to grow at a faster rate, and if its projected MARR is high enough it may be a wiser investment than the corporation. The topic of risk will be discussed in more detail at the end of this chapter.

A reasonable MARR may be determined by considering all these factors (as well as others which may arise). The most important points to remember about the MARR are that:

1. it is expressed as a percentage
2. it acts as the **discount rate** for all cash flows associated with the project (the concept of discount rate is discussed in Chapter 3)

Applying the MARR as a discount rate allows you to make an estimate of the present value of a project's cash flows which accounts for opportunity costs, the cost of capital, and other important factors. Setting an appropriate MARR is a key step in evaluating investment opportunities and ultimately making smart financial decisions.





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## 5.2 Basic Evaluation Methods

### 5.2.1 Conventional Payback Period Method

When deciding on whether to make an investment, people will often ask “How long will it take to get my money back?” This is what the payback period method allows us to determine.

The conventional payback period refers to the period of time required for the return on an investment to “repay” the sum of the original investment. Put another way, the conventional payback period is the amount of time it takes to get back as much money as you invested. Payback period is most often calculated in years. For a one-time investment that generates a uniform series of revenues the payback period ( $n$ ) is calculated as follows:

$$n = \frac{\text{initial investment}}{\text{revenues per time period}}$$

Note that  $n$  expressed in the same time units as the (recurring) revenues (e.g. years).

**WARNING:** this simplified equation is only applicable for a single investment with a uniform series of cash flows; more complex cash flows would require each cash flow to be considered separately.

When making decisions, people may use the payback period as a criterion by setting a benchmark time within which they want to earn their money back. If the payback period occurs before the cut-off point, the project may be accepted; otherwise, it will be rejected. In this way, the payback period can help inform investment decisions in a manner similar to the MARR.

Let’s try some examples.

#### Payback Period Example #1

Alex is considering purchasing a new espresso machine which can make a cup of espresso cheaper than he can buy at his local coffee shop. The machine costs \$360, and he expects it to save him \$60 per year. What is the conventional payback period of this purchase?

Since this example involves uniform cash flows, we can use the simplified payback period equation.

$$\text{Initial investment} = \$360$$

Revenues per time period = \$60/year

$$n = \frac{\text{intital investment}}{\text{revenues per time period}} = \frac{\$360}{\$60/\text{yr}} = 6 \text{ years}$$

The conventional payback period of Alex's new espresso machine is 6 years.

## Payback Period Example #2

Brandon is considering purchasing a similar espresso machine for \$375. However, Brandon is a first year engineering student, and expects that his field of study will impact his coffee consumption. If the new machine saves him \$75 for each of his first three years, \$100 during his fourth (and final) year of school, and \$50 per year after he has graduated, what is the conventional payback period of this purchase?

### Solution

Unlike the previous example, we are not dealing with a uniform series of cash flows in this example, so we cannot use our simplified equation. We solve it instead by creating a table showing cash outflow and inflows from Brandon's perspective:

Year	Cash Flow	Cumulative Cash Flow	How we calculated the cumulative amount....
0	-\$375.00	-\$375.00	
1	\$75.00	-\$300.00	(-375+75=-300)
2	\$75.00	-\$225.00	(-300+75=-225)
3	\$75.00	-\$150.00	(-225+75=-150)
4	\$100.00	-\$50.00	(-150+100=-50)
5	\$50.00	\$0.00	(-50+50=0)
6	\$50.00	\$50.00	(0+50=50)

By looking at the cumulative cash flows, we can see that the initial investment is earned back at the end of year 5. At this point, the cash outflow and inflows were equal. Therefore, the conventional payback period in this scenario is 5 years.

Sometimes, like in these two examples, the payback period is easy to see because they give a cumulative cash flow of exactly \$0 at the end of one of the periods.

This is not true of all cases, however. Sometimes, the cash flows will equal zero part way through a year (or other

time period), so the payback period will not be a whole number. In these cases, we typically assume that the cash flows occur continuously throughout the time period (i.e. not just in lump sums at the end of each period). For example, if a project brings in \$365 per year, we assume that it makes \$1 per day. This allows us to use linear interpolation to estimate where the payback point occurs within the time period.

Let's look at an example that requires linear interpolation.

### Payback Period Example #3

Carla is another first year engineering student, with coffee drinking habits very similar to Brandon from Example #2. She purchases the same espresso machine as Brandon, but finds it on sale for \$345. If she saves the same annual amounts as Brandon, what is the conventional payback period of her purchase?

#### Solution

Let's look at the cash outflows from Carla's perspective:

Year (x)	Cash Flow	Cumulative Cash Flow (y)
0	-\$345.00	-\$345.00
1	\$75.00	-\$270.00
2	\$75.00	-\$195.00
3	\$75.00	-\$120.00
4	\$100.00	-\$20.00
5	\$50.00	\$30.00
6	\$50.00	\$80.00

Here we see that we cannot determine the exact payback period by just looking at the cash flows because Carla's cumulative cash flow is not exactly zero at the end of any year. We can see, however, that the cumulative cash flow is negative at the end of year 4, but positive at the end of year 5. This means that the payback period must have occurred sometime between the 4th and 5th years. We can approximate the actual time by using linear interpolation. Linear interpolation is a reasonable method in this case, because we can assume that the cash flows reflect small amounts saved each day on cups of espresso not purchased at a coffee shop.

Since we know the payback must occur between years 4 and 5, we use those as our known points. Setting years as our x-values and cumulative cash flow as our y-values, we have the two points:

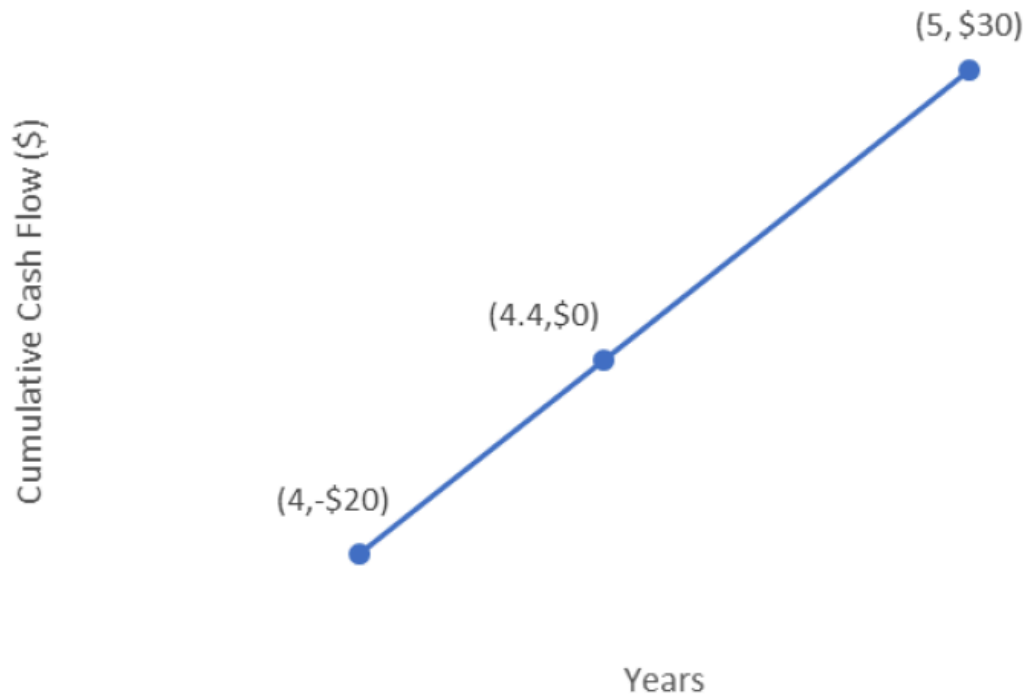
$$(x_0, y_0) = (4, -\$20)$$

$$(x_1, y_1) = (5, \$30)$$

We now interpolate between these points to find the value of  $x$  for which  $y = \$0$ . Using this information, we solve for  $x$ :

$$\frac{y-y_0}{y_1-y_0} = \frac{x-x_0}{x_1-x_0}$$

$$x = \frac{y-y_0}{y_1-y_0}(x_1 - x_0) + x_0 = \frac{\$0 - (-\$20)}{\$30 - (-\$20)}(5 - 4) + 4 = 4.4 \text{ years}$$



You can see in the diagram above how linear interpolation finds the point at which our cash flows are equal to zero.

Therefore, the conventional payback period for this project is 4.4 years.

**Gut check:** Is this reasonable? Carla paid less than Brandon, so her payback period should be less than his. Her payback period of 4.4 years is shorter than Brandon's payback period of 5 years, so this answer is reasonable.

### Advantages & Drawbacks of the Conventional Payback Period Method



Advantages	Disadvantages
<ul style="list-style-type: none"> <li>• Easily understood</li> <li>• Widely used</li> <li>• Favours liquidity</li> <li>• Effectively measures investment risk</li> </ul>	<ul style="list-style-type: none"> <li>• No consideration of TVM</li> <li>• Ignores long-term cash flows</li> <li>• Ignores project risk</li> <li>• Arbitrary cut-off point</li> <li>• Biased toward short term projects</li> </ul>

## ADVANTAGES

- **Easily understood:** Investors often want to know how long it will take to get their money back if they choose to invest. The payback period gives this information and can be understood regardless of academic or technical background.
- **Widely used:** Members of many industries use and understand this concept, allowing for straightforward communication between parties regarding investment opportunities.
- **Favours liquidity:** By choosing projects with shorter payback periods, a company quickly earns back the nominal value of its investment, and can invest in other projects sooner.
- **Effectively measures investment risk:** The project with a shortest payback period typically has less risk than with the project with longer payback period. (Boundless Finance)

## DRAWBACKS

- **No consideration of TVM:** Since the conventional payback period method only uses the nominal values of future cash flows, it overstates the values of those cash flows because it does not account for the effect of time on their value, as discussed in Chapter 3. This becomes particularly significant for projects involving high interest rates or long time periods. For this reason, it is generally recommended that this method not be the sole criterion for decision making.
- **Ignores long-term cash flows:** This method does not consider any cash flows after the payback period. For example, many long-term construction projects require site cleanup costs or disposal costs after a project is finished. Also, any revenues obtained after the payback period are not considered, despite the fact that they may be substantial. The conventional payback period analysis ignores both of these factors because they occur after the project's revenues have "paid off" the initial expenditure.
- **Ignores project risk:** The expected future revenues or expenses of the project used in the calculations are considered certain, and no analysis of potential risks to the project is considered.
- **Arbitrary cut-off point:** The conventional payback method forces the decision maker to set a strict payback point. However, there is no real reason to define a specific payback period unless there are known events that will occur beyond that period which would require you to be paid back. For example, if you know your company will need a large amount of capital available in 4 years to expand its head office, then it might make sense to expect a payback period of less than 4 years for any major

projects.

- **Biased toward short term projects:** By emphasizing quick payback, this method downplays the importance of long-term planning.

## 5.2.2 Net Present Value (NPV) Analysis

The net present value (NPV) method is one of the simplest and most reliable ways to decide whether or not to undertake a project or investment. The strength of this method is its consideration of the time value of money (covered in Chapter 3).

The NPV is the sum of the present values of all costs and revenues associated with the project.

NPV is calculated using the following formula:

$$NPV = \sum_{n=0}^N \frac{Revenue_n}{(1+i)^n} - \sum_{n=0}^N \frac{Cost_n}{(1+i)^n}$$

Notice that that equation is the present value formula, but summing multiple cash flows.

The net present value of an investment or project can be understood as the value that you could receive today that would be economically equivalent to undertaking the project. That is, accepting a \$100 bill today would be economically equivalent to undertaking a project whose NPV was \$100.

By calculating the NPV, we can determine whether a project is economically profitable. If the NPV is a positive number, then the project is profitable. If the NPV is a negative number, then the opposite is true: the project is not profitable and should not be accepted (if profit is the only goal of the project). If the NPV is at least zero, then the project meets the specified MARR. Note that in reality, a project may be accepted or rejected based on other factors, such as public image.

Sometimes, you may have the opportunity to choose from several different projects, but can only choose one and must reject all others. In this situation, we say the projects are mutually exclusive. The NPV method can help us decide which project to accept. When given multiple mutually exclusive projects, the one with the highest NPV should be accepted. Note, however, that if all the projects have negative NPVs, we should likely still reject all of them.

To calculate the NPV of a project, we need some initial information:

- The discount rate of the project. This rate is usually the company's MARR.
- The expected timing and amounts of all cash flows of the project. More accurate estimates of these values will give a more accurate NPV.

To refresh your concept of present value and the time value of money, consider the following example:

Erica sees that a local lottery is advertising a \$1.5 million dollar prize, so she buys a ticket, and ends up winning. After winning, the lottery company contacts her and informs her that her \$1.5 million dollar prize will be awarded over the course of 10 years, with a \$150,000 payment each year beginning today, rather than as a lump sum today.

Erica is overjoyed at having won, so she accepts these terms without considering the time value of money. If her local bank offers an interest rate of 5%, how much more preferable would the lump sum payment have been?

Since a dollar today is worth more than a dollar tomorrow, it will be better for Erica to have her money up front. To see this difference, we must find the present value of the ten-year series of payments, and compare it to the \$1.5 million lump sum.

We will show the present value calculations explicitly for the first three payments, and then show the remainder of the process in a table. Note that the first payment is received immediately, and so its present value will still be equal to \$150,000.

$$P_0 = \frac{F_1}{(1+i)^n} = \frac{\$150,000}{(1+0.05)^0} = \$150,000.00$$

$$P_1 = \frac{F_1}{(1+i)^n} = \frac{\$150,000}{(1+0.05)^1} = \$142,857.14$$

$$P_2 = \frac{F_1}{(1+i)^n} = \frac{\$150,000}{(1+0.05)^2} = \$136,054.42$$

Year of Cash Flow	Present Value
0	\$150,000.00
1	\$142,857.14
2	\$136,054.42
3	\$129,575.64
4	\$123,405.37
5	\$117,528.92
6	\$111,932.31
7	\$106,602.20
8	\$101,525.90
9	\$96,691.34

**Sum (NPV)                      \$1,216,173.25**

Thus, the NPV of a project is \$1,216,173.25. Since the NPV of a lump sum payment is \$1.5 million, the difference

is:

$$\$1,500,000.00 - \$1,216,173.25 = \$283,826.75$$

So, although Erica will still receive \$1.5 million (the nominal amount), the economic (real) value of her prize is \$283,826.75 less because it is paid over 10 years.

### ***The Do-Nothing Alternative***

In many cases, investment is not required, and the **do-nothing alternative** exists. The term “do-nothing alternative” can mean two distinct things:

- A) Do not invest in any alternatives and do nothing; or,
- B) Do not invest in any alternatives and continue as is.

In Case A, all cash flows are zero. There are no investments, revenues, or costs. This simply means that if we are presented with several projects to choose from and none of them are desirable, we can decline all of the options. However, in Case B (also known as the status quo), no new investment is made and the costs and revenues are merely those being incurred under current operations.

Examples do-nothing alternatives include:

- You are considering starting a business, and have estimated your costs and revenues. If the NPV is negative, you should not start the business. This option of doing nothing results in neither a financial gain nor a financial loss (Case A).
- A corporation considering acquiring a competing business. If the acquisition appears to be unprofitable, then the corporation would not pursue the acquisition and continue operating as is (Case B).

However, the do-nothing alternative is not always available. Some examples of situations in which the do-nothing alternative is not available include:

- A company deciding to replace something vital to its business, such as a Coca-Cola bottling plant replacing its bottle-filling machines as they wear out.
- A car dealership owner deciding what vehicle models to order and sell from his supplier.
- A growing city upgrading its sewer system and wastewater treatment plant.

In these cases, the decision maker cannot choose the do-nothing alternative. Put another way, they must choose at least one of the options being considered. If the bottling plant does not replace its equipment as it wears out, eventually the machines will no longer be functional, and the business will have to shut down production. Similarly, if the car dealership does not order new vehicles as it sells the vehicles in its inventory, it will run out vehicles and cease to operate. If the city does not expand its sewage management infrastructure... you get the idea.

When the do-nothing alternative is not available, economic analyses only need to compare investment/purchase/project opportunities against each other; for example, the bottling plant simply needs to select which bottle-filling machine is most profitable (or least expensive).

Below is a list of pros and cons of the NPV method.

Advantages	Drawbacks
<ul style="list-style-type: none"> <li>• Easily understood</li> <li>• Widely used</li> <li>• Favours wealth maximization</li> <li>• Considers TVM</li> <li>• Easily adjusted by discount rate</li> </ul>	<ul style="list-style-type: none"> <li>• Highly sensitive to discount rate</li> <li>• Requires future cash flow estimates</li> <li>• Ignores opportunity cost</li> <li>• Requires identical project duration</li> </ul>

## ADVANTAGES

- **Easily understood:** Investors want to know which potential investments will provide them with the most economic value. The NPV method gives this information and can be understood regardless of academic or technical background (Boundless Finance).
- **Widely used:** Members of many industries use and understand this method, allowing for straightforward communication between parties regarding investment opportunities.
- **Favours wealth maximization:** The NPV method measures the size of the profits to be made, unlike some methods which only measure the relative profitability of the project.

For example, suppose you have two mutually exclusive projects: A and B. The project details are given in a table.

Project	Initial Cash Outlay (\$)	NPV of Entire Project (\$)
A	1	10
B	100	500

In this case, some other methods, such as the IRR method, would declare that project A is the better option because it has a higher return on investment. However, if the projects are mutually exclusive and you cannot repeat the same project it is clear from the NPV method that project B should be chosen because it maximizes profits. In the NPV method, we do not care about the efficiency of investments – only wealth maximization.

- **Easily understood:** The NPV method is easy to understand because it answers a simple question: what should I invest in to get the highest profits for myself or my company? Net present value analyses are easy to explain to audiences who may not know much about the mathematics of finance. This is very

important in industry, where investment decisions often require approval from a manager or board of directors who are not familiar with the projects themselves.

- **Considers TVM:** By discounting all future cash flows and comparing them in a common timeframe, the net present value method allows us to calculate the true economic value of cash flows in a project or investment. This, of course, leads to better financial decisions than if the time value of money were ignored, as it is in analysis methods such as the payback period method.
- **Easily adjusted by discount rate:** NPV is very sensitive to the discount rate used in the calculation. This means that if you are skeptical of a project's viability, or uncertain about the value and timing of the cash flows, you can raise the interest rate to see if the project is still profitable.

## DRAWBACKS

- **Highly sensitive to discount rate:** NPV analysis is essentially just discounting cash flows, which makes it highly sensitive to the discount rate chosen. This means that choosing the correct discount rate is important in NPV analysis. However, it is often difficult to know exactly what discount rate is appropriate, so assumptions must be made when choosing a rate. Basing the discount rate on insufficient information and large assumptions increases the error in NPV calculations.
- **Requires future cash flow estimates:** NPV analysis requires reasonable estimates of all future cash flows – both their timing and their amounts. Often, accurate estimates of future cash flows are difficult to obtain. For example, when deciding whether to purchase a new piece of equipment, it may be hard to predict annual maintenance costs or cost savings. The more uncertain we are about the revenues and costs of the investment, the more error in the NPV calculation.
- **Ignores opportunity cost:** “Furthermore, the NPV is only useful for comparing projects at the same time; it does not fully build in opportunity cost. For example, the day after the company makes a decision about which investment to undertake based on NPV, it may discover there is a new option that offers a superior NPV. Thus, investors don't simply pick the option with the highest NPV; they may pass on all options because they think another, better, option may come along in the future. NPV does not build in the opportunity cost of not having the capital to spend on future investment options.”(Boundless Finance)
- **Requires identical project duration:** The only way to compare projects of different durations using the NPV method is to analyze them over a period that is equal to the least common multiple of the individual projects' periods.

When the do-nothing alternative is available, it can be considered in one of two ways:

A) By evaluating it as an explicit option like any other project, but with all revenues and costs equal to zero

or

B) By measuring all the other alternatives against the do-nothing alternative, and subtracting its cash flows from the option being considered.

### NPV Example

You own a ceramics manufacturing company. The ovens your plant currently uses are expected to last another 8 years. You are considering replacing your ovens with a newer, more efficient model that saves heating costs. You estimate that outfitting your plant with the new ovens would cost \$3 million, the ovens would last 8 years, and the move would save the company \$400 000 per year, starting next year. If your company's MARR is 4%, is the project worth undertaking?

### Solution

In order to determine whether the project is worth undertaking, we can simply determine the net present value of the cash flows.

$$N = 8 \text{ years}$$

$$A = \$400\,000$$

$$i = 4\% = 0.04$$

$$NPV = P_{cost} + P_{savings} = -\$3,000,000 + \$400,000 \frac{1.04^8 - 1}{0.04(1.04)^8} = -\$306,902.05$$

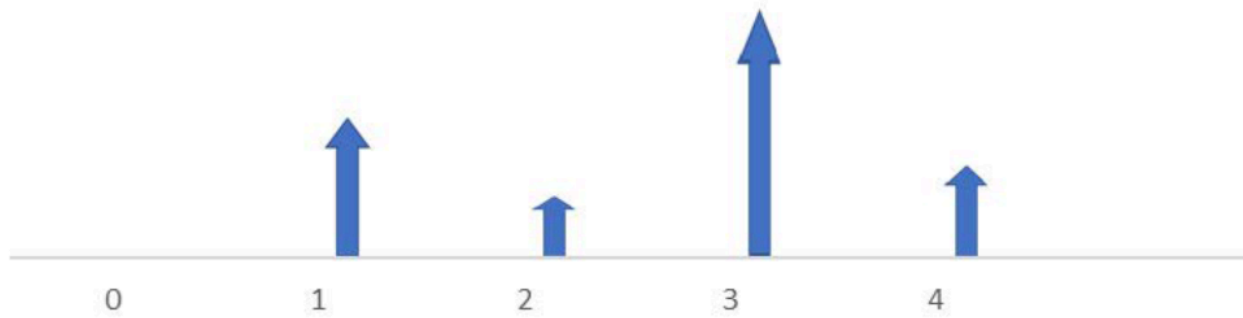
Since the NPV of the project is negative, the **project should be rejected**.

### 5.2.3 Annual Equivalent Value (AEV) Analysis

Rather than considering how much a project will earn us overall using the net present value (NPV) method, we may prefer to know how much it will earn us over time. For this, we use the **annual equivalent value (AEV)** method, which converts the NPV of our project into an economically equivalent, uniform series of cash flows. This allows us to visualize how much our project will earn us (or cost us) each year. For example, this may be useful when considering two projects with an equivalent NPV, but different durations; the shorter project earns money in a shorter time frame, and therefore would have a higher AEV.

To visualize this process, consider the following three cash flow diagrams. All of these diagrams are **economically equivalent**; that is, they all have the same NPV. The first diagram shows an unequal series of cash flows, like you might receive from a 4-year project. The second diagram converts these cash flows into a single cash flow in Year 0, which is the NPV of the project. The final diagram converts this NPV to an annual equivalent value, so that each year we see an equal revenue being produced. All of these have the same NPV.

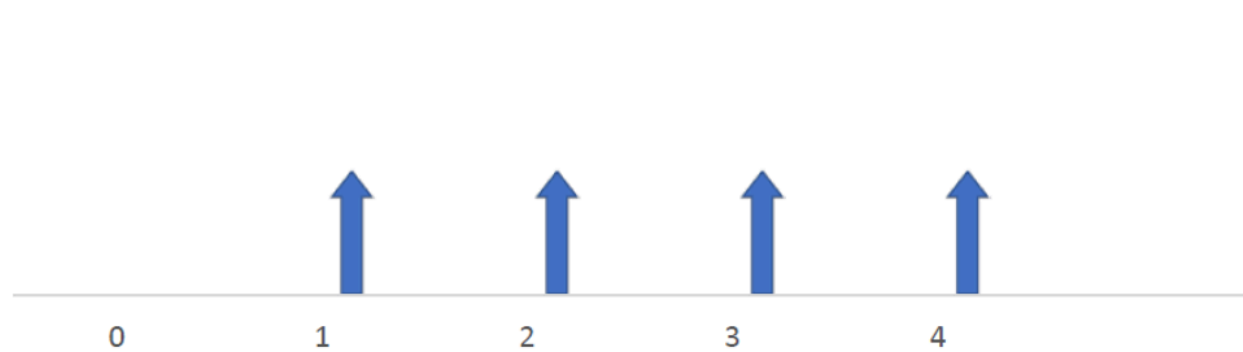
## a) Unequal series of Cash flows



## b) Net Present Value of Cash Flows



## c) Annual Equivalent Value of Cash Flows



The formula to convert a project's NPV to an AEV is:

$$AEV = NPV \frac{i(1+i)^N}{(1+i)^N - 1}$$

Notice that this is an adaptation of the payment-from-present-value formula introduced in Chapter 3. Since AEV analysis is an adaptation of NPV analysis, it has many of the same advantages. In particular, it accounts for the TVM and emphasizes wealth maximization, unlike the conventional payback period method.

#### Advantages & Drawbacks of the Annual Equivalent Value (AEV) Method



Advantages	Drawbacks
<ul style="list-style-type: none"> <li>• Easily understood</li> <li>• Favours wealth maximization</li> <li>• Considers TVM</li> <li>• Easily adjusted by discount rate</li> </ul>	<ul style="list-style-type: none"> <li>• Highly sensitive to discount rate</li> <li>• Requires future cash flow estimates</li> </ul>

## ADVANTAGES

- **Easily understood:** An investor may wish to know how much they will earn from a project on an annual basis. The concept of a consistent annual return on investment can be understood regardless of academic or technical background.
- **Favours wealth maximization:** As with NPV analysis, AEV analysis measures the size of potential profits rather than their relative profitability. This means that it will favour projects which maximize wealth.
- **Considers TVM:** The discount rate applied in AEV analysis (usually MARR) allows for the method to account for the time value of money.
- **Easily adjusted by discount rate:** As with NPV analysis, AEV analysis can be tested at multiple discount rates in order to evaluate its likely profits.

## DRAWBACKS

- **Highly sensitive to discount rate:** Since this method is so easily adjustable by the discount rate, it is also very sensitive to that rate, and a slightly unrealistic rate can produce misleading results.
- **Requires future cash flow estimates:** AEV analysis requires accurate estimates of the timing and amount of future cash flows; the more inaccurate the estimates, the more inaccurate the AEV will be.

## AEV Example

You work for a potash mine that plans to temporarily expand its processing facilities for a cost of \$2,000,000. This expansion will increase profits by an estimated \$740 000 annually. Maintaining the new facilities will cost \$20,000 during the first year of operations, and this cost will increase by \$1500 each year as the equipment deteriorates. If the expansion is put in place for a period of 4 years and the mine applies an MARR of 10%, what is the annual equivalent value?

## Solution

1 – Begin by finding the NPV of the initial cost (1), annual profits (2), and annual operating costs (3). Notice that the increased profits represent a linear series, while the cost of operations is a gradient series.

$$i = 10\% = 0.1$$

$N = 4$  years

$$NPV_1 = -\$2,000,000$$

$$NPV_2 = A \frac{(1+i)^N - 1}{i(1+i)^N} = \$740,000 \frac{(1+0.1)^4 - 1}{0.1(1+0.1)^4} = \$2,345,700.43$$

$$\begin{aligned} NPV_3 &= A \frac{(1+i)^N - 1}{i(1+i)^N} + G \frac{(1+i)^N - iN - 1}{i^2(1+i)^N} \\ &= -\$20,000 \frac{(1+0.1)^4 - 1}{0.1(1+0.1)^4} - 1500 \frac{(1+0.1)^4 - 0.1(4) - 1}{0.1^2(1+0.1)^4} \\ &= -\$63,397.31 - \$6567.17 = -\$69,964.48 \end{aligned}$$

2 – Sum the NPVs to find the total net present value of the project.

$$NPV = NPV_1 + NPV_2 + NPV_3 = -\$2,000,000 + \$2,345,700.43 - \$69,964.48$$

$$NPV = \$275,735.95$$

3 – Convert this NPV to an annual equivalent value (AEV)

$$AEV = NPV \frac{i(1+i)^N}{(1+i)^N - 1} = \$275,735.95 \frac{0.1(1+0.1)^4}{(1+0.1)^4 - 1} = \$89,986.64$$

The Annual Equivalent Value of the expansion is \$86,986.64.

## 5.2.4 Capital Recovery Cost (CRC) Analysis

The capital recovery cost (CRC) method is useful when evaluating an investment that can be salvaged or resold at the end of its project duration. CRC analysis adapts AEV analysis by considering the initial price and future salvage value of the investment, and combining them as an **annual equivalent cost**. This means that the CRC is an estimate of the average cost a company would pay per year to operate a project, when only the initial cost (P) and salvage value (S) are considered:

$$CRC(i_{MARR}) = P \frac{i(1+i)^N}{(1+i)^N - 1} - S \frac{i}{(1+i)^N - 1}$$

You should notice that the first term of the equation, which accounts for the initial price of the investment is derived from the annual given present value equation. Similarly, the second term dealing with the project's salvage value is derived from the equation for annual given future value. Both of these equations are explained in Chapter 3.

The value of this method is to show a company how much it must earn from an investment per year in order for it to be profitable. If a project is expected to generate less annual revenue than its CRC each year, then it should be rejected. Note that the formula for CRC incorporates the company's MARR as its discount rate, so that a project which earns more than its CRC overcomes the company's expectations for opportunity cost, inflation, cost of

capital, and risk. It should also be noted that in some cases CRC can be an insufficient basis for the financial strength of a project, since it does not consider any ongoing costs associated with the project beyond its initial price.

### Advantages & Drawbacks of the Capital Recovery Cost (CRC) Method

Advantages	Drawbacks
<ul style="list-style-type: none"> <li>• Favours wealth maximization</li> <li>• Considers TVM</li> <li>• Easily adjusted by discount rate</li> </ul>	<ul style="list-style-type: none"> <li>• Highly sensitive to discount rate</li> <li>• Requires salvage value estimate</li> </ul>

### ADVANTAGES

- **Favours wealth maximization:** As with NPV and AEV analyses, CRC analysis measures the size of potential profits rather than their relative profitability. This means that it will favour projects which maximize wealth.
- **Considers TVM:** The discount rate applied in CRC analysis (usually MARR) allows for the method to account for the time value of money.
- **Easily adjusted by discount rate:** As with NPV and AEV analyses, CRC analysis can be tested at multiple discount rates in order to evaluate its likely profits.

### DRAWBACKS

- **Highly sensitive to discount rate:** Since this method is so easily adjustable by the discount rate, it is also very sensitive to that rate, and a slightly unrealistic rate can produce misleading results.
- **Requires salvage value estimate:** CRC analysis requires an accurate estimate of the salvage value at the end of a project, which may be difficult to predict.

### CRC Example

Noah's company buys him a new pickup truck for conducting field surveys. The truck costs \$32 000, and after 5 years of use the company plans to resell it for \$24,000. If Noah's company uses an MARR of 8%, what is the Capital Recovery Cost (CRC) of this investment?

### Solution

$$P = \$32,000$$

$$S = \$24,000$$

$$i = 8\% = 0.08$$

$N = 5$  years

1 – Apply the standard capital recovery cost formula

$$CRC(i_{MARR}) = P \left( \frac{i(1+i)^N}{(1+i)^N - 1} \right) - S \left( \frac{i}{(1+i)^N - 1} \right)$$

$$CRC(i_{MARR}) = \$32,000 \left( \frac{0.08(1+0.08)^5}{(1+0.08)^5 - 1} \right) - \$24,000 \left( \frac{0.08}{(1+0.08)^5 - 1} \right)$$

The capital recovery cost of the new truck is \$3923.66

Remember that this value is equivalent to the annual cost of owning the truck, and this investment must generate over \$3923.66 per year for it to be a worthwhile investment.

### 5.2.5 Internal Rate of Return (IRR) Method

As discussed at the beginning of this chapter, the rate of return for a project is a percentage showing the total profits divided by the investment cost. This concept allowed us to consider the minimum acceptable rate of return (MARR), which could be used as a discount rate when determining the viability of a project. In order to compare the return on our project directly to our MARR, we can calculate the **internal rate of return (IRR)**.

The IRR is defined as the discount rate (usually annual) that gives a project a net present value of zero. In more specific terms, the IRR of an investment is the discount rate at which the net present value of the costs (negative cash flows) of the investment equals the net present value of the benefits (positive cash flows) of the investment. The term “internal” refers to the fact that its calculation does not incorporate environmental factors (e.g., the interest rate or inflation).

IRR calculations are commonly used to evaluate the desirability of investments or projects. If a project’s IRR exceeds a company’s MARR, then it is worth investment. The higher its IRR, the more desirable it is to undertake the project. Why? Let’s do a couple of thought experiments.

1. There are two projects, each of which costs \$1000. Project A returns \$1400 after two years, and Project B returns \$1500 after two years. Which of these would you prefer?
2. There are two projects, each of which return \$1600 after two years. Project C costs \$1400 up front, and Project D costs \$1500 up front. Which of these would you prefer?

In these scenarios, it is fairly straightforward to determine which project is economically preferable. Now, let’s use these questions to show how IRR can be used to make decisions.

1 – There are two projects, each of which costs \$1000. Project A returns \$1400 after two years, and Project B returns \$1500 after two years. Which of these has a higher IRR?

We will calculate the internal rate of return for each by setting our NPV equation equal to zero:

$$NPV_A = P + \frac{F}{(1+i_A)^N}$$

$$0 = -1000 + \frac{1400}{(1+i_A)^2}$$

$$i_A = 18.3\%$$

$$NPV_B = P + \frac{F}{(1+i_B)^N}$$

$$0 = -1000 + \frac{1500}{(1+i_B)^2}$$

$$i_B = 22.5\%$$

We see that Project B has a higher IRR than Project A because it produces a greater return for the same initial cost, and thus would be the better investment.

2 – There are two projects, each of which return \$1600 after two years. Project C costs \$1400 up front, and Project D costs \$1500 up front. Which of these has a higher IRR?

$$NPV_C = P + \frac{F}{(1+i_C)^N}$$

$$0 = -1400 + \frac{1600}{(1+i_C)^2}$$

$$i_C = 6.9\%$$

$$NPV_D = P + \frac{F}{(1+i_D)^N}$$

$$0 = -1500 + \frac{1600}{(1+i_D)^2}$$

$$i_D = 3.3\%$$

We see that Project C has a higher IRR than Project D because it produces the same return for a lower initial cost, and thus would be the better investment. Notice that both of these rates of return are fairly low; if you had a MARR of 8%, you would choose to reject both of these projects.

As we can see from these examples, the project with the highest IRR should be considered the best and undertaken first. A firm (or individual) should, in theory, undertake all projects or investments available with IRRs that exceed their MARR. That is, if the IRR is greater than the MARR, accept the project; if the IRR is less than the MARR, reject the project. In practice, a company might not accept all projects with an IRR exceeding the MARR because they may not have enough money or resources available to do so.

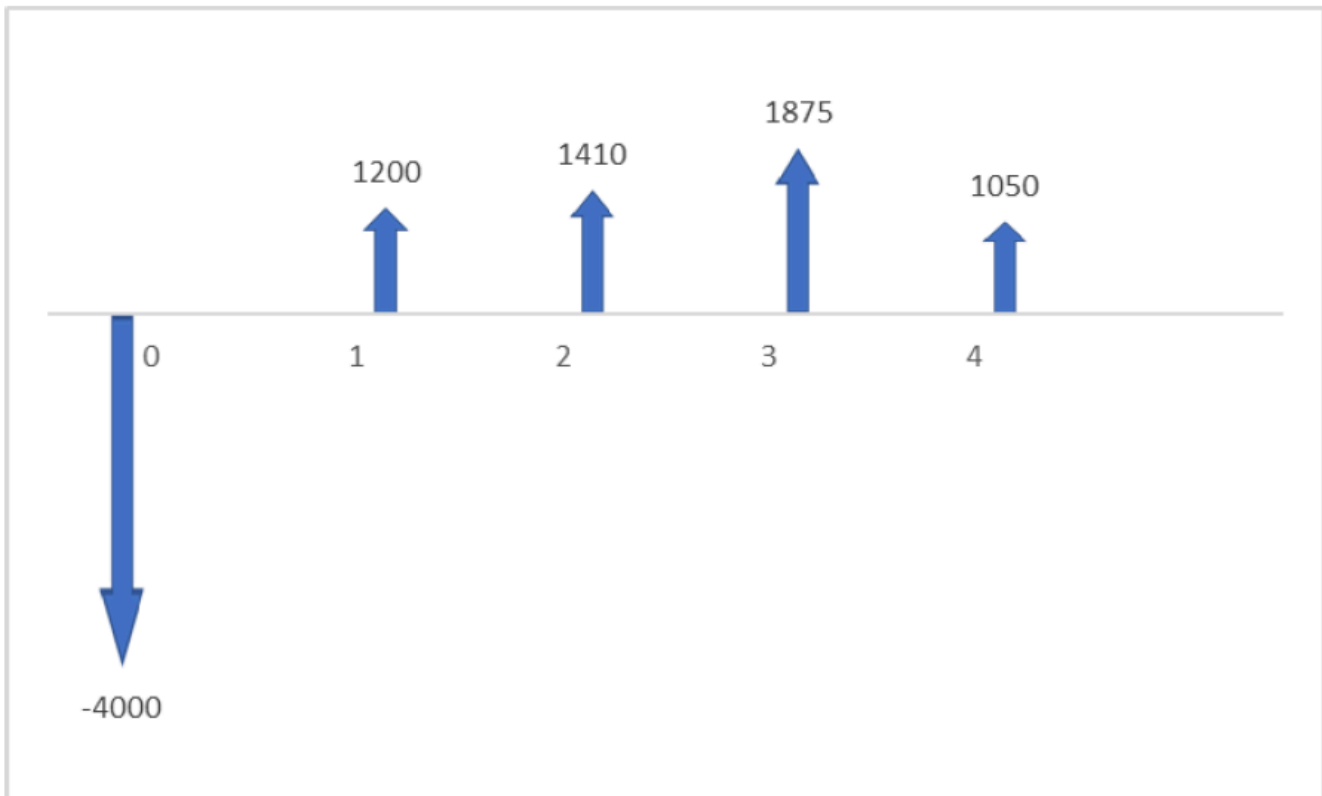
You may wonder: are decisions based on IRR similar to decisions based on NPV? When a project's IRR is equal to its MARR, it will have an NPV of zero. If we approve projects with an IRR greater than our MARR, then we are approving all projects with positive NPVs; this means that *the NPV and IRR methods will agree on whether an individual project should be accepted*. However, determining whether a project yields a profit is different from determining which projects are **more profitable**. When evaluating several mutually exclusive projects, the IRR method may favour a different project than the NPV method. This is because IRR maximizes investment **efficiency** (money earned per dollar spent), whereas the NPV method maximizes **wealth** (net money earned). So, the evaluation method you select will depend on your investment priorities.

### Calculating the IRR

Given a set of cash flows occurring at different times during a project, the internal rate of return can be found by setting the net present value equation equal to zero and solving for 'i'. As always, the time units must be consistent

for each cash flow (e.g. weeks, months, years).

For example, if an investment may be given by the following sequence of cash flows:



Step 1: Write the NPV equation for this series of cash flows, and set it equal to zero.

$$NPV = P + \frac{F_1}{(1+i)^{N_1}} + \frac{F_2}{(1+i)^{N_2}} + \frac{F_3}{(1+i)^{N_3}} + \frac{F_4}{(1+i)^{N_4}} = 0$$

$$NPV = -4000 + \frac{1200}{(1+i)^1} + \frac{1410}{(1+i)^2} + \frac{1875}{(1+i)^3} + \frac{1050}{(1+i)^4} = 0$$

Step 2: Solve for “i”.

In this case, the answer is 14.3%. You may have noticed that this equation is not easy (okay, let’s admit it: basically impossible) to solve by hand. So, how did we arrive at this answer?

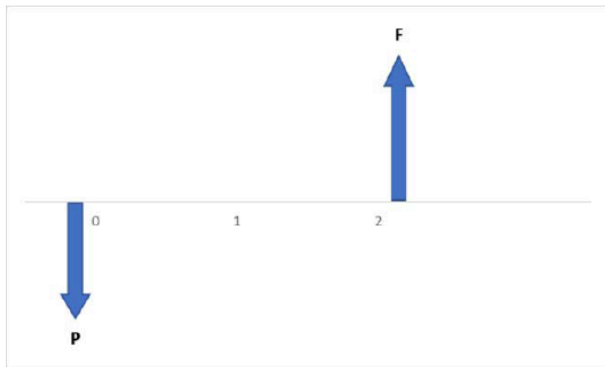
### 5.2.5.1 Methods for Calculating the IRR

Let’s look at three methods for calculating a project’s IRR:

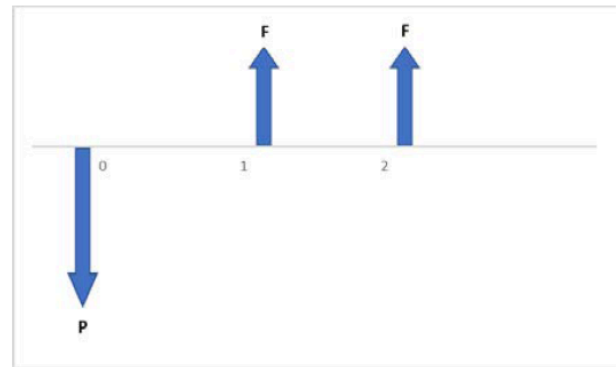
- Direct solution
- Trial and error
- Graphical solution

#### **Method 1: Direct Solution**

The direct solution method is essentially only practical in two cases: A) when the project has only one cash outflow and one cash inflow, and B) when the project has one cash outflow and a return for two periods.



*Case A*



*Case B*

### Case A

In this case, direct solution is fairly straightforward using the net present value formula. We will show this in the two examples below.

#### IRR Example 1

Assume you spend \$2000 today, and receive a return of \$3500 in 4 years. What is the internal rate of return for this investment?

Solution

Since this is a problem with only one cash inflow and one cash outflow, we can isolate the internal rate of return,  $i$ , and solve for its value using our known information (the cash flow amounts and timing) by setting the NPV equation equal to zero.

$$P = -\$2000$$

$$F = \$3500$$

$$N = 4 \text{ years}$$

$$NPV = P + \frac{F}{(1+i)^N}$$

$$NPV = 0 = -2000 + \frac{3500}{(1+i)^4}$$

$$3500 = 2000(1+i)^4$$

$$1.75 = (1+i)^4$$

$$i = \sqrt[4]{1.75} - 1$$

$$i = 15.02\%$$

So the IRR for this project is 15%.

### IRR Example #2

Assume you spend \$2000 today, and you receive returns of \$1300 after 1 year and an additional \$1500 after 2 years. What is the rate of return for this investment?

### Solution

This problem involves three cash flows instead of two, but may still be solved using the direct solution method.

Step 1: Write out the project's NPV as a series of cash flows, and set equal to zero:

$$NPV = 0 = -2000 + \frac{1300}{1+i} + \frac{1500}{(1+i)^2}$$

Step 2: Solve for  $i$ . In this case, we cannot isolate  $i$  because of the exponent of the rightmost term. However, we can make this easier by using a substitution:

$$\text{let } x = \frac{1}{1+i}$$

So:

$$NPV = 0 = -2000 + 1300x + 1500x^2$$

Now we can solve for  $x$  using the quadratic equation.

$$x = \frac{-1300 \pm \sqrt{1300^2 - 4(1500)(-2000)}}{2(1500)}$$

$$x = 0.8 \text{ or } -1.667$$

Now that we have the values of  $x$ , we can use them to calculate  $i$ .

$$0.8 = \frac{1}{1+i} \rightarrow i = 25\% \text{ OR } -1.667 = \frac{1}{1+i} \rightarrow i = -160\%$$

As you can see, we obtain two possible answers for  $i$ . So which one is correct? In this case, we can assume that the IRR is 25%, since an IRR of -160% would imply that the project had cost more than 100% of the initial investment price; this is impossible, since all subsequent cash flows were positive. **So the IRR is 25%.** However, this problem illustrates a shortcoming of the IRR method: there are many situations where mathematical operations will yield two or more possible values for the IRR.

### Method 2: Trial-and-Error

The trial and error method is the process of “guessing” the IRR until the correct value is found. The steps are as follows:

1. Make an initial guess for the internal rate of return (IRR). Calculate the project's NPV using that guess.



2. If the calculated NPV is positive, increase your guess for the IRR and calculate the NPV again. If the NPV is negative, decrease your guess for the IRR and calculate the NPV again.
3. Repeat steps 1 and 2 to continually bring the NPV closer to zero. As the NPV approaches zero, the IRR will approach its true value.

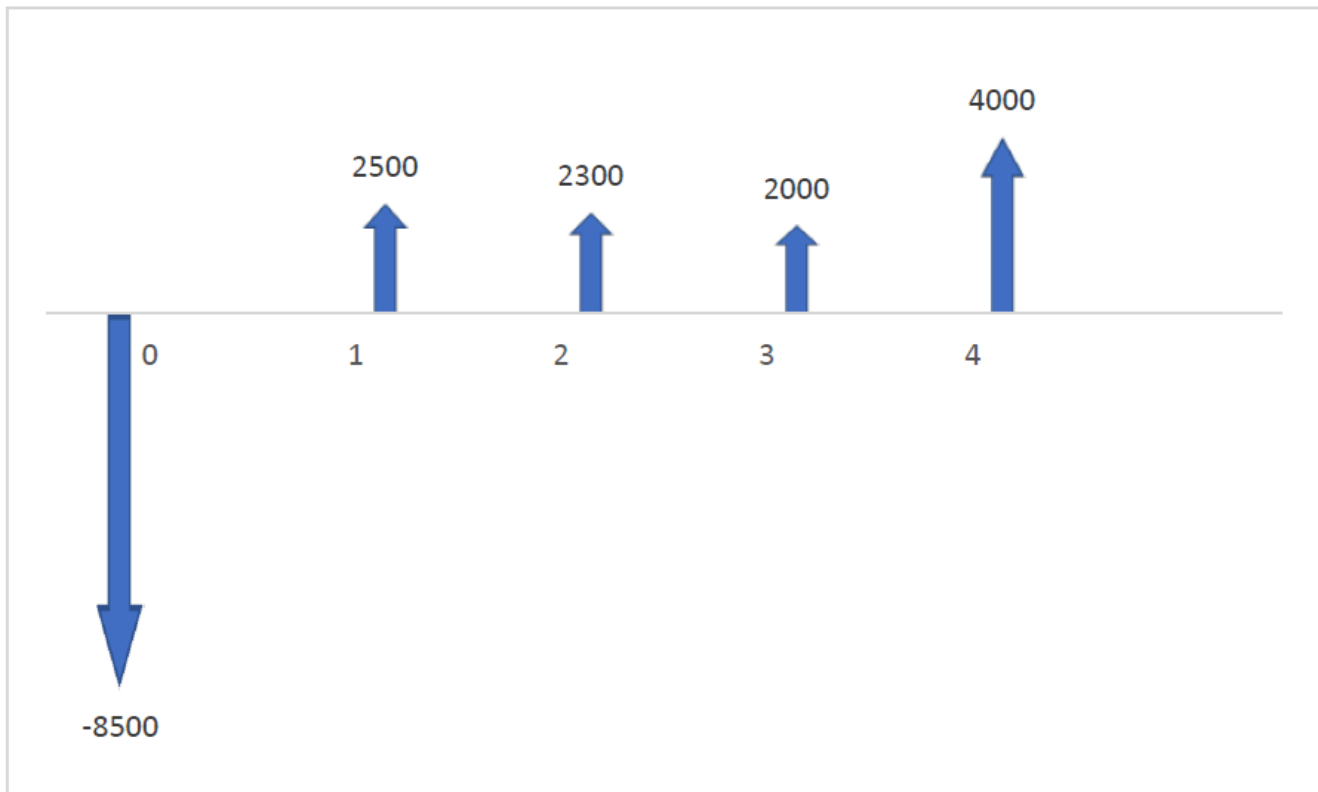
The trial-and-error method can be very tedious to do by hand or with a basic calculator. It is relatively easy to do with a spreadsheet or more advanced calculator.

### Method 3: Graphical Solution

The graphical solution method is similar to the trial and error method. Both calculate multiple values of the NPV by varying the discount rate ( $i$ ). It can also be done easily on a spreadsheet/graphing program. For this method, we select a reasonable range for the rate of return for the project, and then calculate the NPV for these rates. When we graph these data points, they will generate a continuous line graph. And since we know that the IRR of a project is the rate of return which results in a NPV of zero, we look for the point on our line where  $NPV = 0$ . This will give the IRR for the project.

### Example

Suppose you expect the following series of cash flows from a project and want to determine the IRR.



Consider the NPV equation for this series of cash flows:

$$NPV = -8500 + \frac{2500}{1+i} + \frac{2300}{(1+i)^2} + \frac{2000}{(1+i)^3} + \frac{4000}{(1+i)^4}$$

Solving this equation using Method 1 would be very complex and time consuming. Let's try Method 2.

Step 1: Make a guess for the internal rate of return. As a starting point we will guess 5%

Step 2: Calculate the NPV of the project using a discount rate of 5%.

This will generate an answer of \$985.61. Since this value is significantly **positive** we will **increase** our guess for IRR substantially to 10% and try again. If our answer had been **negative**, we would have **decreased** our guess.

Iterate this process several times, increasing and decreasing your guess each time as necessary. A series of guesses might look like this:

IRR Guess	NPV	Result
5%	\$985.61	Increase Guess
10%	-\$91.76	Decrease Guess
9%	\$107.51	Increase Guess
9.5%	\$6.94	Increase Guess
9.55%	-\$3.02	Decrease Guess
9.54%	-\$1.02	Decrease Guess
9.53%	\$0.96	Increase Guess

We can see that the IRR for the project is approximately 9.53%, since it reduces the NPV of the project to nearly zero. The exact IRR must lie between 9.53% and 9.54%, but further precision would require many more iterations. If we wished, we could iterate further, and we would find that the exact IRR for the project is 9.53482776347522%. However, this level of precision is not useful for our purposes.

To check our answer, let's solve the question again using Method 3.

Select a range of possible values which might contain the IRR. For example, suppose we looked at values between 12% and 16%. If the IRR is in this range (i.e. NPV is equal to zero at one point between these two values), then the NPVs of 12% and 16% will have opposite signs. If we calculate NPV for these two values using the equation applied in Method 2, we get the following results:

$$NPV(12\%) = -\$468.68$$

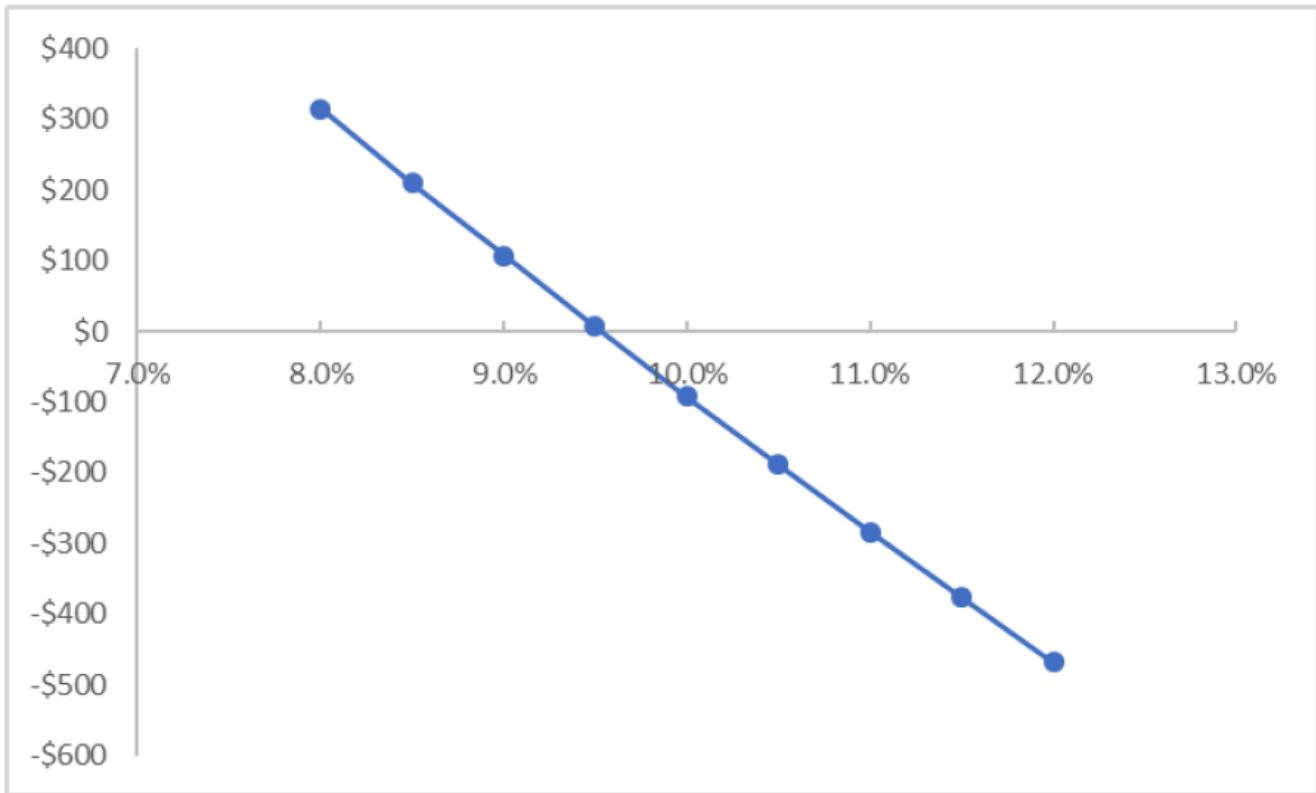
$$NPV(16\%) = -\$1145.07$$

Since both of these results are negative, we know that the IRR does not occur in this range. If we guess a lower discount rate, like 8%, we get an NPV of \$314.48. Since this is a positive value, we know that the IRR must be between 8% and 12%, so we can search for it in this range.

Calculate the NPV of the project for possible IRRs within the range, using the same NPV equation as was used with Method 2. For this example, we will evaluate NPV at 0.5% increments. This will produce the following results:

IRR	NPV
8.0%	\$314.48
8.5%	\$210.01
9.0%	\$107.51
9.5%	\$6.94
10.0%	-\$91.76
10.5%	-\$188.63
11.0%	-\$283.71
11.5%	-\$377.05
12.0%	-\$468.68

Graph the pairs of IRR and NPV values, and find the intercept where NPV is equal to zero: this is the IRR of the project.



We can see that the x-axis intercept of this graph (where NPV=0) is slightly greater than 9.5%; if we were to measure it exactly, we would see that it represents an IRR of 9.53%, the same answer we obtained using the trial and error method.

### Advantages & Drawbacks of the Internal Rate of Return (IRR) Method

Advantages	Drawbacks
<ul style="list-style-type: none"><li>• Easily understood</li><li>• Widely used</li><li>• Favours efficient investments</li></ul>	<ul style="list-style-type: none"><li>• Cannot compare mutually exclusive projects</li><li>• Requires future cash flow estimates</li><li>• Requires identical project durations</li><li>• May produce multiple values</li></ul>

### ADVANTAGES

- Easily understood: Investors will often be interested in knowing the rate of return on a project. Projects with the highest IRRs should be preferred. This makes the consideration process straightforward in a way which is clear and relatable to all decision makers for a project.
- Widely used: The IRR method is commonly used in industry, and will be familiar to most individuals responsible for approving projects.
- Favours efficient investments: Since IRR is a rate percentage, it is an indicator of the efficiency,

quality, or yield of an investment. This is in contrast with the NPV and AEV methods, which are indicators of the value or magnitude of an investment. Higher IRRs indicate higher profits per dollar invested.

## DRAWBACKS

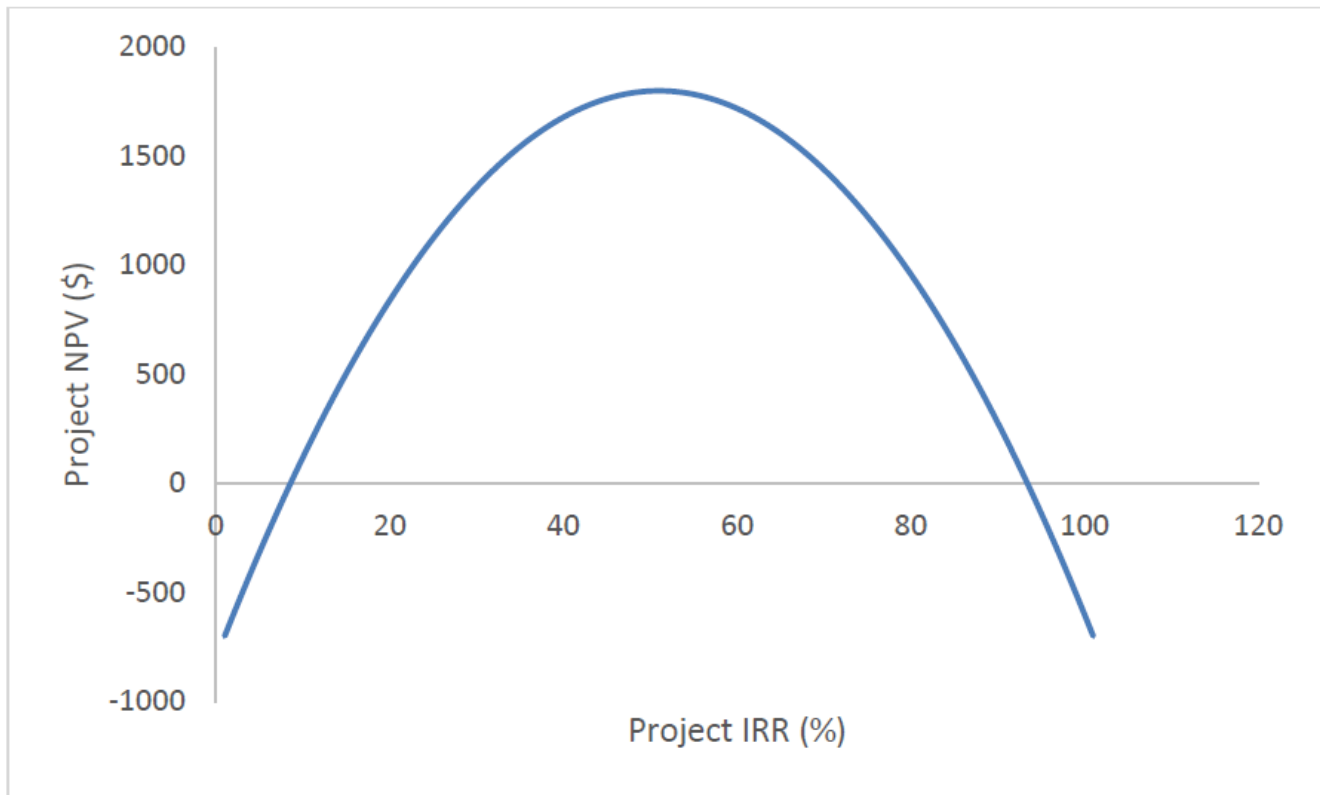
- Cannot compare mutually exclusive projects: IRR should not be used to rate mutually exclusive projects, but only to decide whether a single project is worth investing in. In cases where one project has a higher initial investment than a second mutually exclusive project, the first project may have a lower IRR (expected return), but a higher NPV (increase in shareholders' wealth) and should thus be accepted over the second project (assuming no capital constraints).
- Requires future cash flow estimates: IRR analysis requires accurate estimates of the timing and amount of future cash flows; the more inaccurate the estimates, the more inaccurate the calculated IRR will be.
- Requires identical project durations: Since IRR does not consider cost of capital, it should not be used to compare projects with different durations. Modified Internal Rate of Return (MIRR) does consider cost of capital by applying an MARR, and provides a better indication of a project's efficiency in contributing to the firm's discounted cash flow.
- May produce multiple values: In the case of positive cash flows followed by negative ones and then by positive ones, the IRR may have multiple values.

### 5.2.6 MIRR: An Alternative to the IRR Method

Although the IRR method remains widely used, some issues with the method led to the development of a modified, alternative rate of return method. To understand this alternative method, we should first consider two of the key drawbacks to IRR calculations: a) multiple IRRs, and b) reinvestment assumptions.

#### a) Multiple IRRs

In Figure 5.4 (Method 3) the graph plotting the NPV in relation to the IRR of the project yielded a straight linear function, so there was only one point, or intercept where the NPV was equal to zero, and thus only one IRR value. However, sometimes the relationship between a project's NPV and IRR is non-linear, and there are multiple intercepts representing potential IRR values. We can see an example of this in the NPV-IRR graph below (Figure 5.5), where potential IRR values appear at both 8% and 92%.



This type of result occurs when the cash flows relating to a project alternate between positive and negative more than once over the time period being studied. The more times these values alternate, the more potential IRR values will be generated, and the more confusing it can become to determine the true IRR of a project.

#### b) Reinvestment of Cash Flows

The second major drawback of the IRR method is that it assumes that any revenues produced by the project can be reinvested at a rate equal to the project's IRR. For example, assume we have a 10-year project with a calculated IRR of 25% and in Year 2, the project produced a revenue of \$1000. In order for the project to actually have an IRR of 25%, this \$1000 revenue would need to be reinvested in a new project that also had a 25% rate of return. However, suppose that the next best project available to us only has an IRR of 15%. If we invest our \$1000 revenue in this less lucrative project, then our original project will have an effective rate of return lower than 25%, and the IRR method will have overestimated its value.

We can deduce that this must be a common problem with the IRR method because we will probably invest in the most profitable project first, so any subsequent projects we can reinvest in are very likely to have lower IRRs. As a result, we will likely overestimate the benefit of our first project.

This brings us to the Modified Internal Rate of Return...

The **modified internal rate of return** (MIRR) method addresses both of these problems. First, it groups together all negative and positive cash flows to eliminate the possibility of multiple IRRs. Second, it assumes that all revenues are reinvested at your MARR rather than at the IRR of the project, which prevents overestimation of the

IRR due to unrealistic reinvestment assumptions.

There are three steps to finding the MIRR of a project:

1. Convert all negative cash flows over the project duration to a single negative cash flow at the start of the project using the NPV formula. Use your MARR as the discount rate.
2. Convert all positive cash flows over the project duration to a single positive cash flow at the end of the project using the future value formula. Again, use your MARR as the discount rate.
3. Using the present value equation, substitute the negative cash flow as the present value (P), and the positive cash flow as the future value (F), and solve for the discount rate (i). This is the rate which makes the positive and negative cash flows equal, and thus is the MIRR of your project.

The MIRR is evaluated in the same way as an IRR: if the MIRR exceeds your MARR, then the project is a worthwhile investment; if it is less than your MARR, then it isn't a worthwhile investment.

Let's try an example where we can apply the MIRR process.

### MIRR Example

Gillian is a pilot who plans to offer small charter flights to remote northern communities. Her business will be very seasonal, and may not yield a profit every year due to varying demand, weather, and maintenance expenses. She has asked you to invest in her business, based on the projected cash flows for the first five years of operation, shown in the table below. Assuming you have a personal MARR of 8%, use the MIRR method to determine whether you should invest in Gillian's business.

Year	0	1	2	3	4	5
Cash Flow	-\$20,000	\$6,000	-\$2,000	\$8,000	\$1,500	\$14,000

Notice that the cash flows in this example alternate several times between positive and negative values. With the traditional IRR method, this would have produced multiple IRR values, so the MIRR method is a more effective option. Let's follow our 3 step process to find the MIRR of this project.

### Solution

Step 1: Convert all negative cash flows to a single present value. We do this by applying the NPV formula with our MARR as the discount rate to the values for years 2 and 4.

$$NPV = P_1 + \frac{F_1}{(1+i_{MARR})^N} + \frac{F_2}{1+i_{MARR})^N}$$

$$NPV = -20,000 + \frac{-2000}{(1+0.08)^1} + \frac{-1500}{1+0.08)^2}$$

$$NPV = -\$23,137.86$$

Step 2: Convert all positive cash flows to a single future value at the end of the project. We do this by applying the future value formula with our MARR as the discount rate to the values for years 1, 3, and 5 and moving them all to Year 5.

$$F = P_1(1 + i_{MARR})^N + P_2(1 + i_{MARR})^N + P_3(1 + i_{MARR})^N$$

$$F = 6000(1 + 0.08)^4 + 8000(1 + 0.08)^2 + 14,000(1 + 0.08)^0$$

$$F = \$31,494.13$$

Step 3: Set up a present value equation using our net positive and negative values, and solve for the MIRR.

$$P = \frac{F}{(1 + i_{MARR})^N}$$

$$23,137.86 = \frac{31,494.13}{(1 + i_{MARR})^5}$$

$$i_{MARR} = 6.4\%$$

Our analysis concluded that the MIRR for Gillian's business is **6.4%**. Since this is below your personal MARR of 8%, **you should choose not to invest.**



---

## 5.3 Analysis Period and Project Lives

Earlier in this chapter, we used NPV and AEV analysis to evaluate and compare several projects. However, the projects we considered were reasonably straightforward – the project durations were specified, and projects being compared had equal durations. That is, if a company needed equipment for 5 years, we would compare alternatives with 5-year project durations. But what happens when this is not the case?

For example, suppose that a custom 3D-printing company needs to choose the model of 3D printers it will use to fill customer orders. The company wants to plan for the next 10 years, but the two printers they are considering have useful lives of 4 years and 6 years. Even if we know the purchase prices and maintenance costs of the two printer models, how could we evaluate them appropriately? This section will discuss how we can adapt evaluation methods like AEV and NPV to these situations.

But first, here are two key definitions:

- Analysis period (also called the planning horizon or study period) is the length of time over which the project will occur, and over which we must analyze the project's cash flows to make an economic evaluation. In the 3D-printing example the analysis period is 10 years because that is the length of time over which the company wishes to plan its finances.
- Project life (or project duration) is how long the project will last. In the 3D-printing example there are two projects to choose from: one with a project life of 4 years, and another with a project life of 6 years. After the project life has ended, the equipment is no longer useful to the company and must be either replaced or sold. The dollar amount the equipment can be sold for is the salvage value.

If alternatives have:

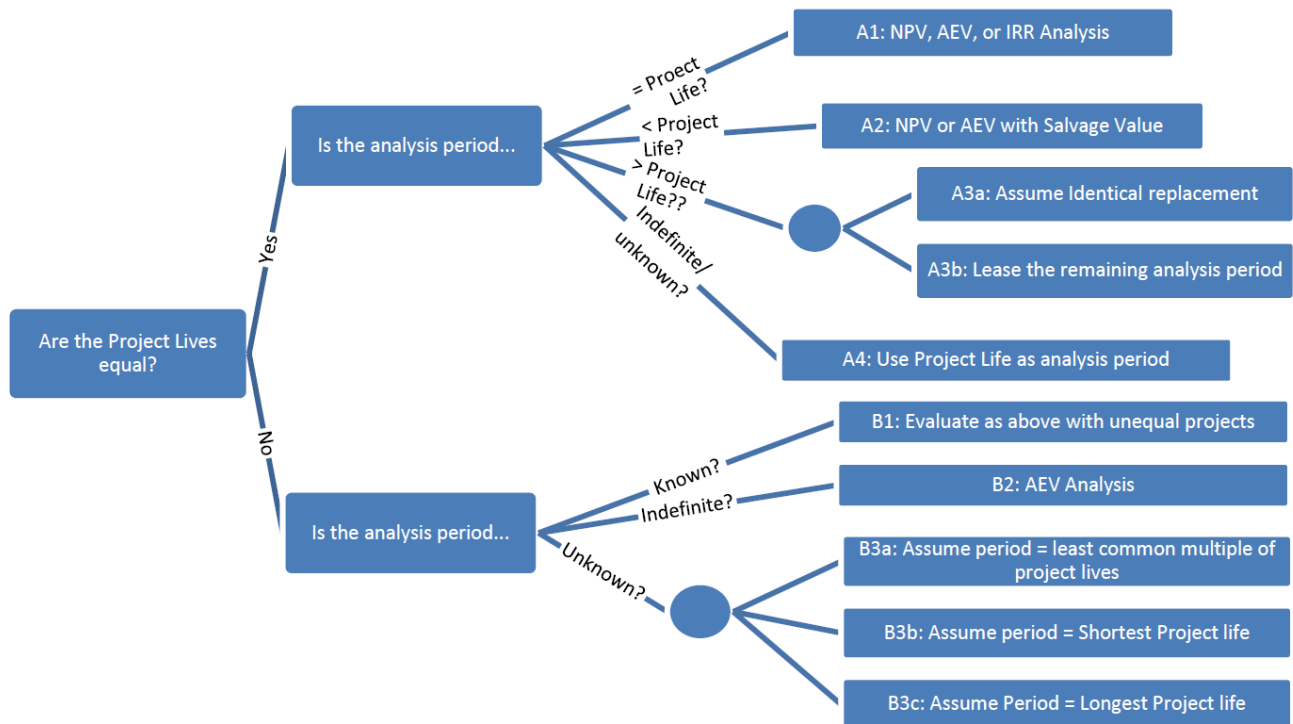
1. unequal project lives, or
2. project lives that are different than the analysis period

These differences need to be accounted for in our analysis. That is, we cannot simply evaluate the alternatives using NPV, AEV, or IRR analysis. We must adjust our methods depending on the situation. To properly analyze these problems, we must first be able to recognize the type of situation we are facing.

There are two important questions to consider for any case we analyze.

1. Do our alternatives have equivalent project lives?
2. Are the project lives equal to our chosen analysis period?

Figure 5.6 below shows the cases we might face depending on our answers to those two questions, and the suggested method of evaluation for each case. These cases and their solutions will each be discussed in greater detail.



## Equivalent Project Lives

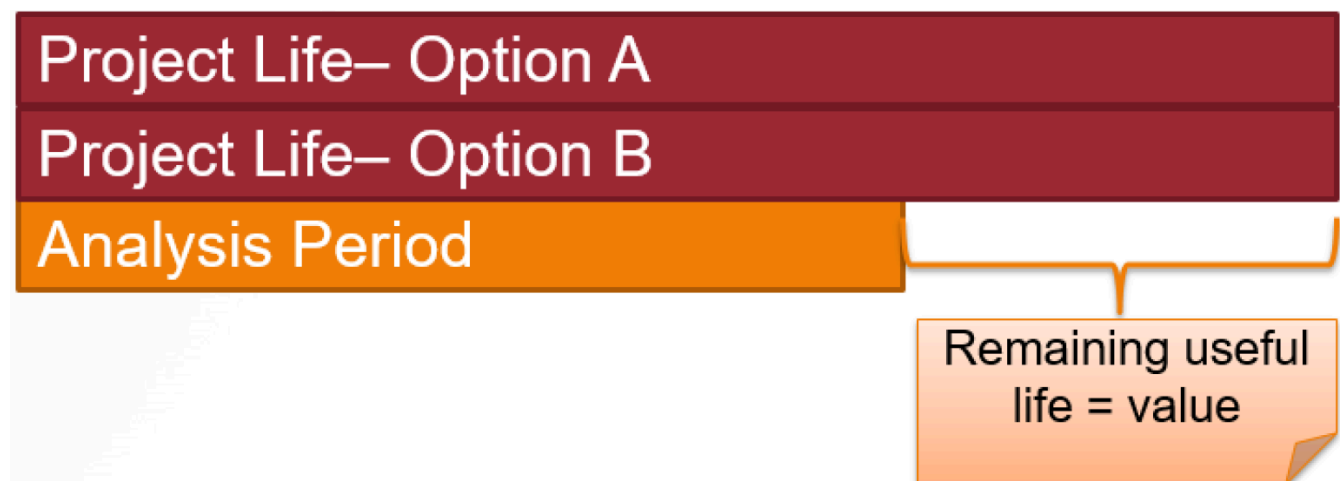
### 5.3.1 A1: Project Lives = Analysis Period

The simplest analysis problems are those in which all project lives are equal to each other and equal to the analysis period. You are already familiar with this type of problem, as the majority of Chapter 5 has exclusively dealt with this scenario. These problems can be solved directly with NPV, AEV, or IRR analysis, and require no special treatment of the cash flows. For more examples of these problems, please refer to the NPV, AEV, or IRR analysis sections.



### 5.3.2 A2: Equal Project Lives > Analysis Period

When all alternatives have equal project lives but last longer than the analysis period, the projects will still have some value remaining at the end of the analysis period. This is the salvage value discussed in the Capital Recovery Cost (CRC) Method. For example: you are trying to choose between two cars which last longer than you need them to, so you sell them at the end of your analysis period and incorporate that salvage value in your analysis. A salvage value is treated as a positive single cash flow occurring at the end of the analysis period, and included in the NPV or AEV analysis like any other cash flow.



Two types of situations may occur: you may be given the salvage value, or it may not be known. First, let's consider an example where the salvage values are given to us.

#### Case 1: Given Salvage Value

In some circumstances, such as with an automobile or a commonplace piece of equipment, it may be possible to estimate a future salvage value with reasonable accuracy. In these situations, we can apply the given salvage value as a single cash flow at the end of the project during our calculations. Consider the following example:

#### Example 1: Given Salvage Value

Mark has rented additional farmland on a 5-year lease, and needs to purchase a new combine harvester to expand his operations. He can either purchase a new combine for \$480,000 or a used combine for \$160,000. The increased profits from the additional combine are unknown, but Mark knows they will be identical regardless of which combine he purchases. He expects the new combine to have maintenance costs of \$2500 per year and sell for \$250,000 after 5 years. He also estimates that the used combine will have annual maintenance costs of \$8000 and sell for \$85,000 after 5 years. If Mark uses a MARR of 12% for his farm, should he buy new or used?

Find the NPV of each combine, including purchase price, maintenance costs, and salvage value. Incorporate the linear series cash flow formula in order to calculate the maintenance costs.

$$NPV_{NEW} = P_{NEW} + A_{NEW} \left[ \frac{(1+i)^N - 1}{i(1+i)^N} \right] + \frac{F_{NEW}}{(1+i)^N}$$

$$NPV_{NEW} = -\$480,000 - \$2500 \left[ \frac{(1+0.12)^5 - 1}{0.12(1+0.12)^5} \right] + \frac{\$250,000}{(1+0.12)^5}$$

$$NPV_{NEW} = -\$347,155.23$$

$$NPV_{USED} = P_{USED} + A_{USED} \left[ \frac{(1+i)^N - 1}{i(1+i)^N} \right] + \frac{F_{USED}}{(1+i)^N}$$

$$NPV_{USED} = -\$160,000 - \$8000 \left[ \frac{(1+0.12)^5 - 1}{0.12(1+0.12)^5} \right] + \frac{\$85,000}{(1+0.12)^5}$$

$$NPV_{USED} = -\$140,606.93$$

Since the used combine has a higher NPV, **Mark should purchase the used machine** for his farm. Note that since Mark's revenues from this new purchase are unknown, it is unclear whether even this lowest cost alternative will be profitable for him.

### Salvage Value Unknown

Sometimes, salvage values are unknown and must be estimated. We assume that the investment (e.g. a piece of equipment) decreases in value by roughly the same amount each year. This allows us to use as the **linear depreciation method**, which is based on linear interpolation, to estimate its salvage value at any point in time.

The formula to find salvage value (S) for a project using this method is shown below, where C is the initial investment cost. (Note that this equation can also be generalized to find the salvage value at any point during the project life)

$$S = C / \frac{\text{Project Life} - \text{Analysis Period}}{\text{Project Life}}$$

While linear depreciation is a common method, it does not apply in all cases. Historical data may show that the depreciation rate is non-linear. One of the most common examples of this case is cars: the value of a new car will decrease dramatically during the first year of ownership, and more gradually in the longer term. We will not do any examples of this kind here, but a similar concept is covered in Chapter 2.4 relating to depreciation.

Let's see an example using the linear depreciation method.

### Example 2: Salvage Value Unknown

A candy manufacturing plant is looking to replace its toffee-pulling machine. Two different companies (Mr. Fantastic & Co. and Stretch Armstrong Inc.) sell basic toffee-pulling machines with the same essential features. The chosen machine will be operated for the next 7 years.

Machine	Initial Cost	Annual Maintenance Costs
Mr. Fantastic	\$10,000	\$1,000
Stretch Armstrong	\$13,000	\$600

Each machine has a useful life of 8 years. If the company's MARR is 9%, which machine should be chosen?

### Solution

In this problem, both models have a project life (useful life) of 8 years, which is longer than the analysis period (7 years). So, we must use salvage values at the end of the analysis period to compare the two models. However, no salvage values are given in the problem statement, so we will estimate their salvage values based on the assumption that their value decreases linearly over time.

To find the salvage values at the end of year 7, we use the equation above.

$$S_{Fantastic} = C_{Fantastic} \left( \frac{\text{Remaining Life}}{\text{Project Life}} \right) = \$10,000 \left( \frac{1 \text{ year}}{8 \text{ years}} \right) = \$1,250.00$$

$$S_{Stretch} = C_{Stretch} \left( \frac{\text{Remaining Life}}{\text{Project Life}} \right) = \$13,000 \left( \frac{1 \text{ year}}{8 \text{ years}} \right) = \$1,625.00$$

Now that we have the salvage values of the two models, we can compare their NPVs. Remember to include the salvage value as a positive cash flow at year 7.

$$NPV = \text{Initial Cost} + P_{\text{maintenance}} + P_{\text{salvage}}$$

$$NPV = P + A \left[ \frac{(1+i)^N - 1}{i(1+i)^N} \right] + \frac{F}{(1+i)^N}$$

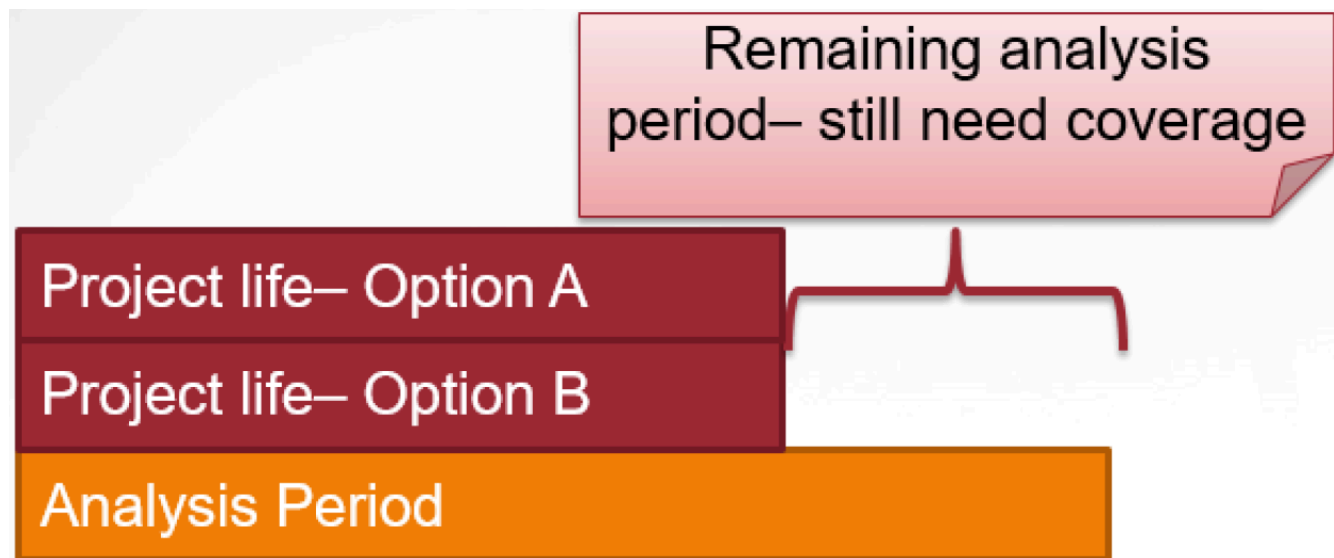
$$NPV_{Fantastic} = -\$10,000 - \$1,000 \left[ \frac{(1+0.09)^7 - 1}{0.09(1+0.09)^7} \right] + \frac{\$1,250}{(1+0.09)^7} = -\$14,349.16$$

$$NPV_{Stretch} = -\$13,000 - \$600 \left[ \frac{(1+0.09)^7 - 1}{0.09(1+0.09)^7} \right] + \frac{\$1,625}{(1+0.09)^7} = -\$15,130.84$$

Because the Mr. Fantastic machine has a higher net present value (that is, it will ultimately cost less than Stretch Armstrong), **Mr. Fantastic should be chosen.**

### 5.3.3 A3: Equal Project Lives < Analysis Period

In some cases, we need to consider several project alternatives whose project lives are equal and shorter than the analysis period. In these cases, the alternatives do not last long enough, so something must be done to cover the remaining portion of the analysis period.



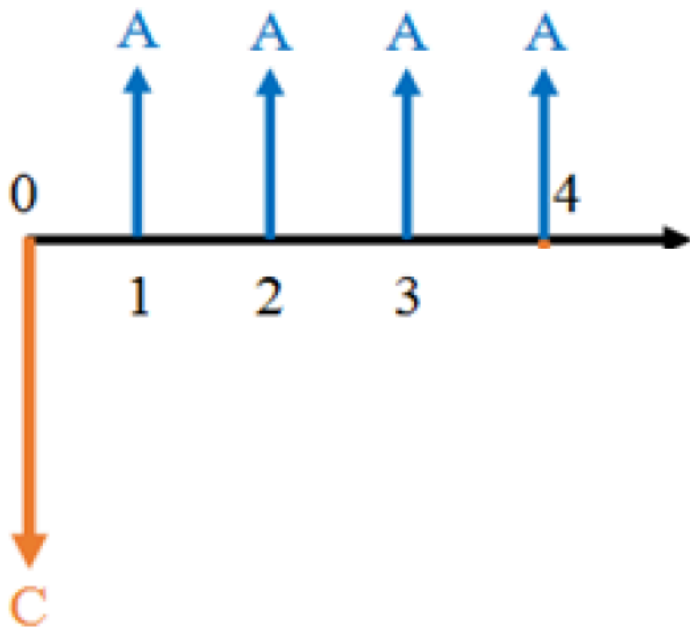
The remaining time in the analysis period can be solved by two methods. You can cover the remaining analysis period by assuming:

1. Each item is replaced with repeated, identical items until the end of the analysis period; or
2. Leasing is used to cover the remainder of the analysis period.

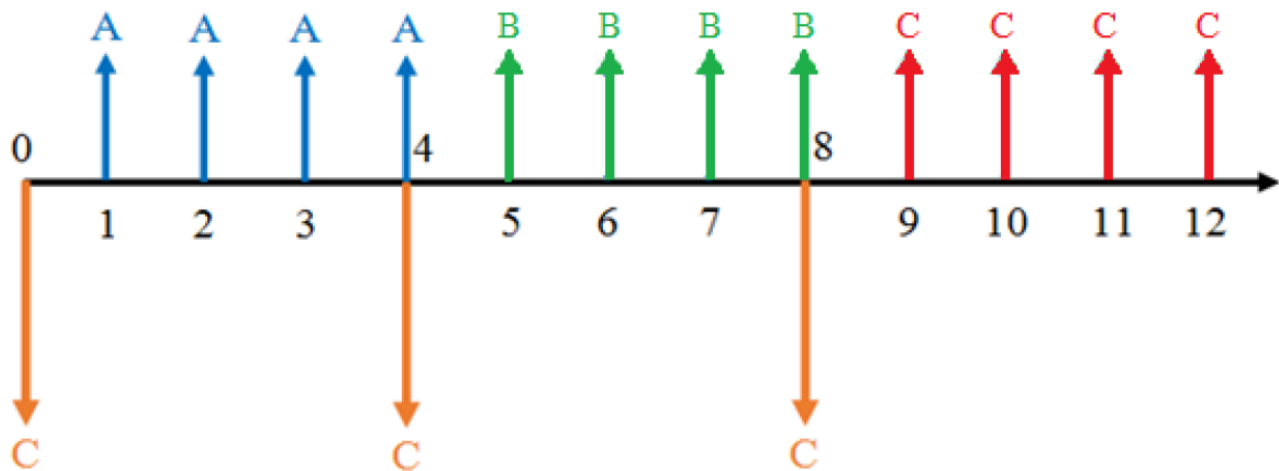
The method chosen to compare an alternative should be applied to all others for consistency. If you use the replacement method for Option A then the replacement method must also be used for Option B.

### **A3a: Repeated Projects**

This method assumes that it is possible to repeat the chosen project with identical cash flows until the end of the analysis period, such as in the illustration below:



The cash flows for Project A, with a four-year project life:

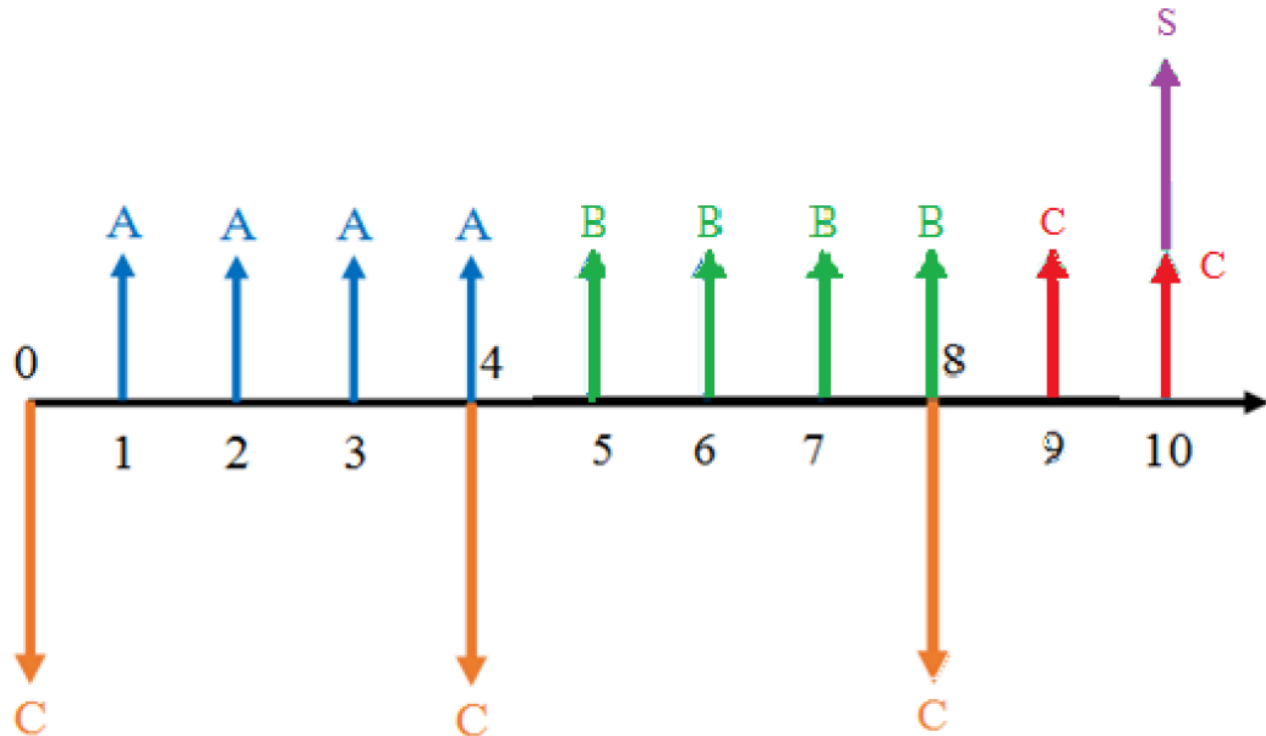


The cash flows for Projects A, B, and C: identical four-year projects repeated over 12 years.

This illustration shows three repetitions of a project that consists of an initial cost followed by equal revenues in the following four years. Each time the project is repeated, the initial cost of the new project must be paid in the last year of the previous project. Consider a taxi driver who replaces their vehicle every three years. They will need to buy a new vehicle at the end of each three year cycle, so they can earn money from that vehicle for each of the following three years.

When repeating projects this way, we can calculate the NPV of the entire set of repeated projects needed to cover the analysis period. We can also use the NPV to obtain an AEV if desired. (Note that the AEV of a set of repeated projects should be the same as the AEV of just one copy of the project.)

Another issue arises, however, in situations where the analysis period is not a multiple of the project life (e.g. the project life is 4 years and the analysis period is 10 years). When the project life does not divide evenly into the analysis period, the last repetition of the project will be cut short – so we use a salvage value! Below is an illustration of the same cash flows as the previous example, but with the salvage value at year 10:



As we can see from the above diagram, an analysis period of 10 years would require 3 copies of a 4-year project. The third copy would be salvaged for its remaining value at the end of the analysis period.

If the salvage value is not given, we can estimate it using the linear formula from the previous section.

### Example

Alyson runs an international shipping company that is adding a new trade route, which requires adding another container ship to their fleet. She must choose between two ships, as shown in the table below: Boaty McBoatFace, which is faster and more expensive, or Shippy McShipFace, which is slower and less expensive.

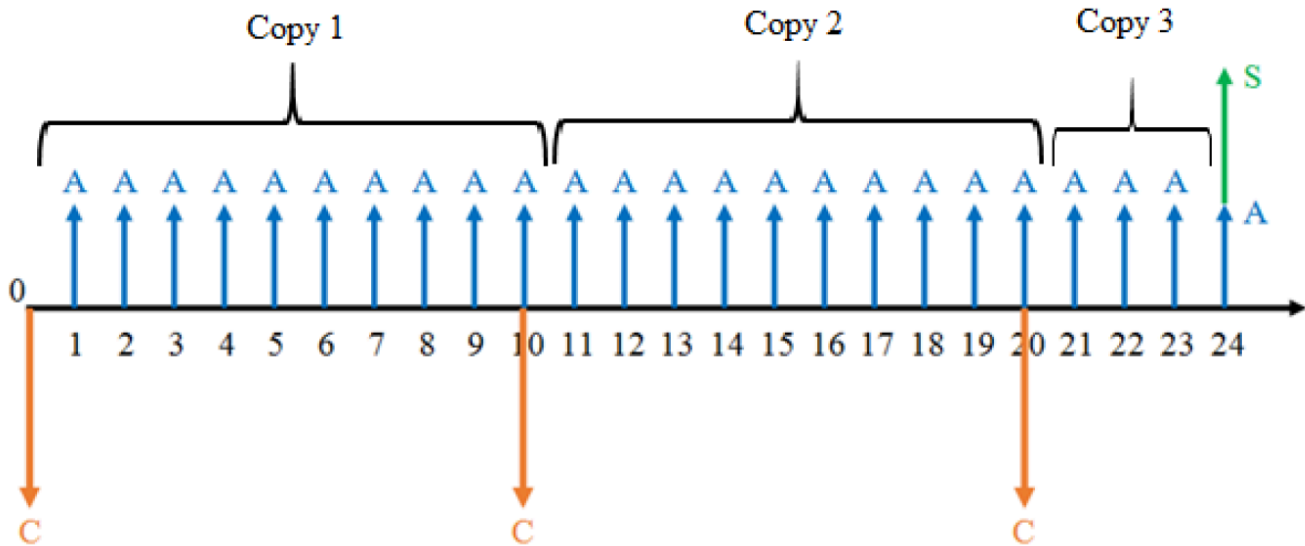
Model	Initial Cost	Annual Earnings
Boaty McBoatFace	\$30 million	\$2 million
Shippy McShipFace	\$20 million	\$1 million

Each model has a useful life of 10 years, after which the salvage cost is cancelled out by the decommissioning cost (i.e. there is zero salvage value). However, she needs to operate this route for the next 24 years. If the company's MARR is 4%, which ship should Alyson choose?



**Solution**

Although both models have useful lives of 10 years, the analysis period for this problem is 24 years. This means that whichever model Alyson chooses, she must use a total of 3 copies of the project (the ship), and salvage the last one at the end of the 24th year – or 4 years into its life. The cash flows for either model would look like this:



1 – Calculate the salvage values that she can expect in the 24th year.

$$S_{Boaty} = C_{Boaty} \left( \frac{\text{Remaining life}}{\text{Project life}} \right) = \$30 \text{ million} \left( \frac{6 \text{ years}}{10 \text{ years}} \right) = \$18 \text{ million}$$

$$S_{Shippy} = C_{Shippy} \left( \frac{\text{Remaining life}}{\text{Project life}} \right) = \$20 \text{ million} \left( \frac{6 \text{ years}}{10 \text{ years}} \right) = \$12 \text{ million}$$

2 – Now that we have the salvage values of the two models, we can compare their NPVs.

$$NPV = P_{Costs} + P_{Earnings} + P_{Salvage}$$

$$NPV = -\$30 \text{ million} \left( 1 + \frac{1}{1.04^{10}} + \frac{1}{1.04^{20}} \right) + \$2 \text{ million} \left[ \frac{1.04^{24} - 1}{0.04(1.04^{24})} \right] + \frac{\$18 \text{ million}}{1.04^{24}}$$

$$NPV = -\$63.96 \text{ million} + \$30.49 \text{ million} + \$7.02 \text{ million} = -\$26.45 \text{ million}$$

$$NPV = -\$20 \text{ million} \left( 1 + \frac{1}{1.04^{10}} + \frac{1}{1.04^{20}} \right) + \$1 \text{ million} \left[ \frac{1.04^{24} - 1}{0.04(1.04^{24})} \right] + \frac{\$12 \text{ million}}{1.04^{24}}$$

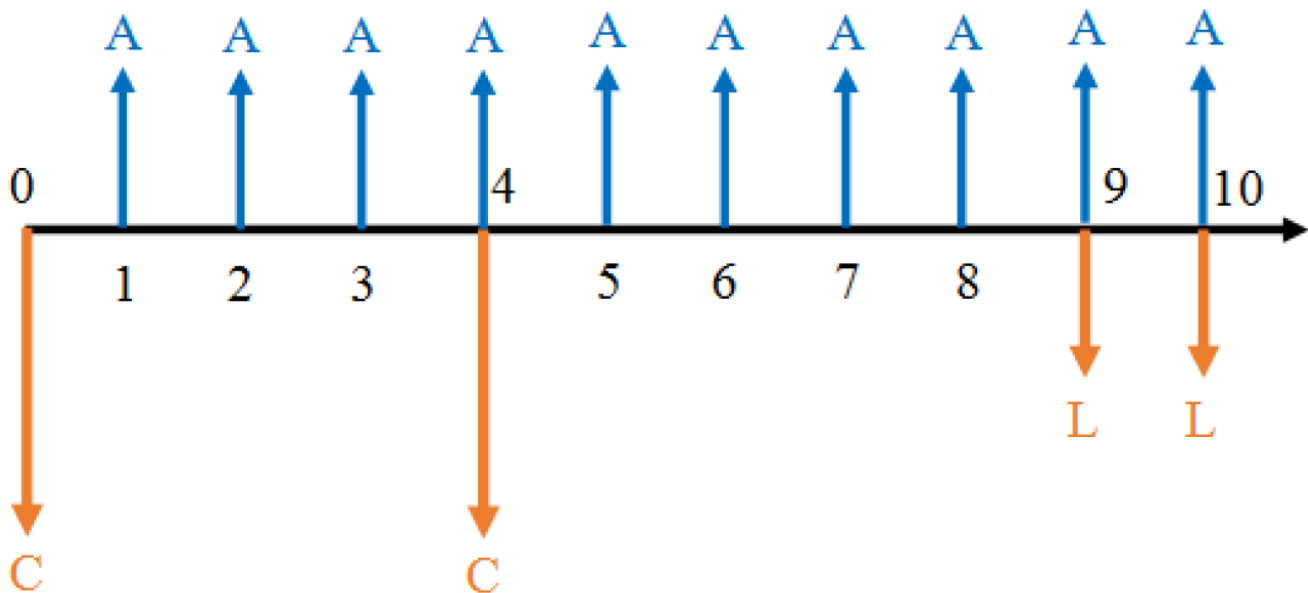
$$NPV = -\$42.64 \text{ million} + \$15.25 \text{ million} + \$4.68 \text{ million} = -\$22.71 \text{ million}$$

Because Shippy McShipFace ultimately has a lower cost, **Shippy McShipFace should be chosen.**

**A3b: Leasing**

When using repeated projects to fill an analysis period, relying on the salvage value of the final project is not always the most economical option. Another way we can ensure coverage for the entire analysis period is by leasing the needed equipment. We still repeat the chosen project as needed, but we only repeat the project whenever we can use its entire useful life. Once the remainder of the analysis period is shorter than the useful life of the chosen alternative, we cover the remainder with leasing.

Consider a project with a 4 year duration being applied over a 10 year analysis period. Using leasing, the cash flows for the scenario would look like this:



In the above diagram, the cash flows labelled ?? represent lease payments for the equipment. As we can see, the project life is 4 years, which does not divide evenly into the 10-year analysis period. Under the leasing method, we use the same project twice, beginning in year 0 and year 4. This covers the first 8 years. Purchasing another project, however, would require salvaging the project after the 10th year. Instead, we use leasing to cover the 9th and 10th years.

#### 5.3.4 A4: Equal Project Lives with Indefinite Analysis Period

For some cases, like a highway or other major infrastructure project, the analysis period for the project may be practically indefinite, or at least unknown. In such cases, we have no defined cut-off point at which to salvage projects or begin a lease. Since we need a baseline for comparing alternatives, **the project life is used as the analysis period**. We can then apply a basic evaluation method such as NPV or AEV, which will reduce the problem to a form that we are now familiar with.

#### Unequal Project Lives

Analysis becomes quite different when at least one alternative has a different useful life than the others. When this is the case, special steps must be taken to ensure the cash flows are evaluated properly.

When project alternatives have unequal lives, there are three analysis cases to consider:

1. the analysis period is known and considered finite, or
2. the analysis period is considered indefinite, or
3. the analysis period is unknown and considered finite

### 5.3.5 B1: Unequal Project Lives with Known Analysis Period

If the analysis period for a project is known, but project lives are unequal, the situation is not significantly different than with equal project lives; however, we must consider the projects separately. This is best shown by considering a simple example:

Your donut shop is considering three mutually exclusive donut-making-machines over an analysis period of 20 years. Machine A costs \$20,000 and will earn \$1600/year over a project life of 25 years. If it is salvaged 5 years early, its salvage value will be \$2000. Machine B costs \$5000 and will earn \$1500/year over a project life of 4 years. It can also be repeated as many times as necessary. Machine C costs \$17,000 and will earn \$1475/year over a project life of 20 years. Assume a MARR of 5%. Which project should you invest in?

Since all three projects have different project lives, we must consider them all separately. We can use the same approaches for each independent project as we did for our equal project lives examples, and afterwards compare the NPV of each project.

For Machine A, the project life is longer than the analysis period, so we can conduct an NPV analysis with an added salvage value (as in A1).

$NPV_A = \text{Initial Price} + \text{Annual Revenue} + \text{Salvage Value}$

$$NPV_A = P_A + A_A \left[ \frac{(1+i)^N - 1}{i(1+i)^N} \right] + \frac{S_A}{(1+i)^N}$$

$$NPV_A = -\$20,000 + \$1600 \left[ \frac{(1+0.05)^{20} - 1}{0.05(1+0.05)^{20}} \right] + \frac{\$2000}{(1+0.05)^{20}}$$

$$NPV_A = \$693.32$$

For Machine B, the project life is shorter than the analysis period, so we consider identical replacement projects for the entire analysis period (as in A2). So first, let's find the NPV of the machine over its 4 year project life:

$$NPV_B = -\$5,000 + \$1500 \left[ \frac{(1+0.05)^4 - 1}{0.05(1+0.05)^4} \right]$$

$$NPV_B = \$318.93$$

To fill the analysis period, we will require 5 identical machines, each contributing \$318.93 every four years (note that the N values substituted in are therefore multiples of 4); we can find the total NPV of this option by considering all 5 machines:

$$NPV_B = \frac{318.93}{(1+0.05)^0} + \frac{318.93}{(1+0.05)^4} + \frac{318.93}{(1+0.05)^8} + \frac{318.93}{(1+0.05)^{12}} + \frac{318.93}{(1+0.05)^{16}} = \$1120.88$$

For Machine C, the project life is equal to the analysis period (as in A3), so we calculate the NPV result as usual:

$$NPV_C = \$17,000 + \$1475 \left[ \frac{(1+0.05)^{20} - 1}{0.05(1+0.05)^{20}} \right]$$

$$NPV_C = \$1,381.76$$

We can see by comparing the NPVs of each project, that Machine C is the most lucrative and should be invested in.

### 5.3.6 B2: Unequal Project Lives with an Indefinite Analysis Period

Sometimes, we do not know exactly how long a project will be used, but we expect that it will be used for a very long time, perhaps indefinitely. This situation often arises with public infrastructure projects such as roads, parks, water distribution networks, Egyptian pyramids, etc. In these scenarios, the cash flows may be known, but we cannot select a reasonable analysis period. To solve these problems, we simply use AEV analysis.

#### Example

Your plastics manufacturing company wants to replace one of its injection moulds. You have two options:

- Mould A that lasts 5 years with an initial price of \$5000, with maintenance costs of \$500/yr
- Mould B that lasts 7 years with an initial price of \$7000, with maintenance costs of \$400/yr

If your company's MARR is 6%, which option is cheaper? Assume that your company plans to be in business long term.

#### Solution

Because the problem statement tells us that the company plans to be in business long term, we assume the chosen mould will be used "indefinitely". Since the project lives are unequal, we will use AEV analysis. Let's calculate the AEV of each alternative, and compare.

Remember that AEV gives an annual amount. If we look at the two mould options, we see that the maintenance costs are already in annual payments! This means that, instead of calculating the NPV of the maintenance costs and then converting to an AEV, we can just add the maintenance costs "as-is" to the AEV of the upfront cost.

#### Mould A

Maintenance costs:  $A_A = -\$500$

Purchase price:  $P_A = -\$5000$

$i = 6\%$

$N = 5$  years

$$AEV_A = P_A \left[ \frac{i(1+i)^N}{(1+i)^N - 1} \right] + A_A = -\$5000 \left[ \frac{0.06(1+0.06)^5}{(1+0.06)^5 - 1} \right] - \$500 = -\$1686.98$$

### Mould B

Maintenance costs:  $A_B = -\$400$

Purchase price:  $P_B = -\$7000$

$i = 6\%$

$N = 7$  years

$$AEV_B = P_B \left[ \frac{i(1+i)^N}{(1+i)^N - 1} \right] + A_B = 7000 \left[ \frac{0.06(1+0.06)^7}{(1+0.06)^7 - 1} \right] - \$400 = -\$1653.95$$

We can see that Mould B has a higher AEV (lower annual cost, in this case). Therefore, **your company should choose Mould B.**

### 5.3.7 B3: Unequal Project Lives with Finite Analysis Period

When the analysis period is considered finite, but is unknown, we must choose what it will be. There three options:

- A) Analysis period = least common multiple (LCM) of project lives
- B) Analysis period = shortest project life among alternatives
- C) Analysis period = longest project life among alternatives

Often, this choice is entirely arbitrary and is based on the preference of the analyst. Note that the choice of analysis period should **not** affect the outcome of the analysis. In some fringe cases this might pose an issue; for example, suppose a winery purchased a very expensive, specialized piece of equipment for sealing casks of wine. Since the equipment is so specialized, there might be no second-hand buyers available, making the salvage value essentially zero. In this case, the equipment might be a valuable solution over 25 years, but extremely inefficient over 5 years compared to another option (e.g. paying labourers to seal the casks by hand) since its up front cost is so high. These sorts of scenarios are rare, however, and most often the same project will be preferable regardless of the chosen analysis period.

Let's solve an example using each of the three methods listed above and compare the results.

### Unequal Project Lives Example

Suppose that a city is replacing its fleet of garbage collection trucks. It is currently choosing between 3 models: the Truckasaurus, the Trashgobbler, and the Garbotron 3000. The city will replace all their trucks with trucks of the chosen model. The table below lists the costs associated with replacing the city's fleet.

Model	Initial Cost	Annual Maintenance Costs	Project Life
Truckasaurus	\$5 million	\$375,000	5 years
Trashgobbler	\$4 million	\$475,000	4 years
Garbotron 3000	\$10 million	\$325,000	10 years

If the city has a MARR of 5% per year, which model should be chosen?

### Solution

Start by listing all given values. We will use  $C$  for initial costs and  $A$  for annual maintenance costs.

Truckasaurus:  $C_A = \$5$  million,  $A_A = \$375,000/\text{yr}$ , Project life = 5 years

Trashgobbler:  $C_B = \$4$  million,  $A_B = \$475,000/\text{yr}$ , Project life = 4 years

Garbatron:  $C_C = \$10$  million,  $A_C = \$325,000/\text{yr}$ , Project life = 10 years

$i = 5\%$  per year for all options

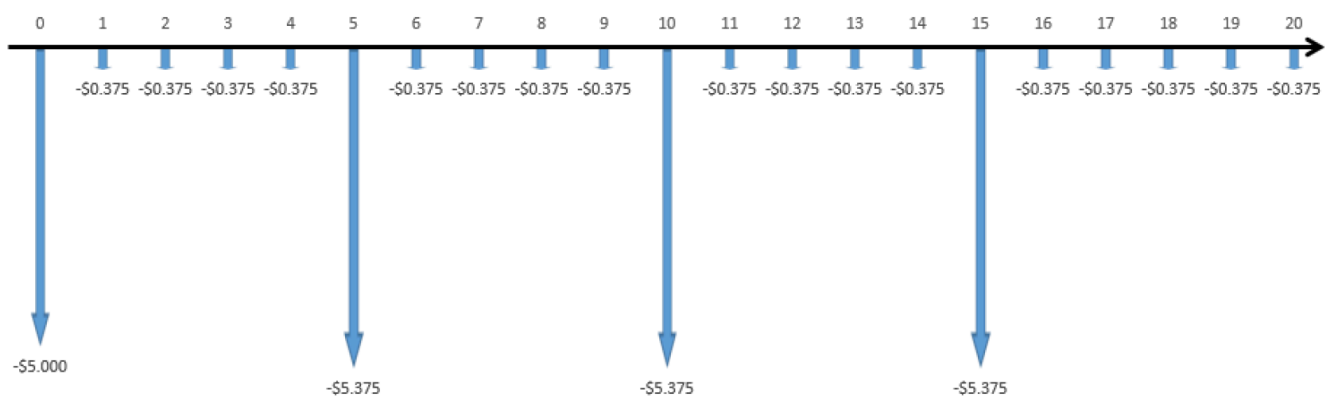
The discount rate and the maintenance costs have the same time units, so we can proceed. However, all project lives are unequal, and the analysis period is unknown. Since we have to choose an analysis period, for this example we will perform the analysis three times, using each of the above methods once.

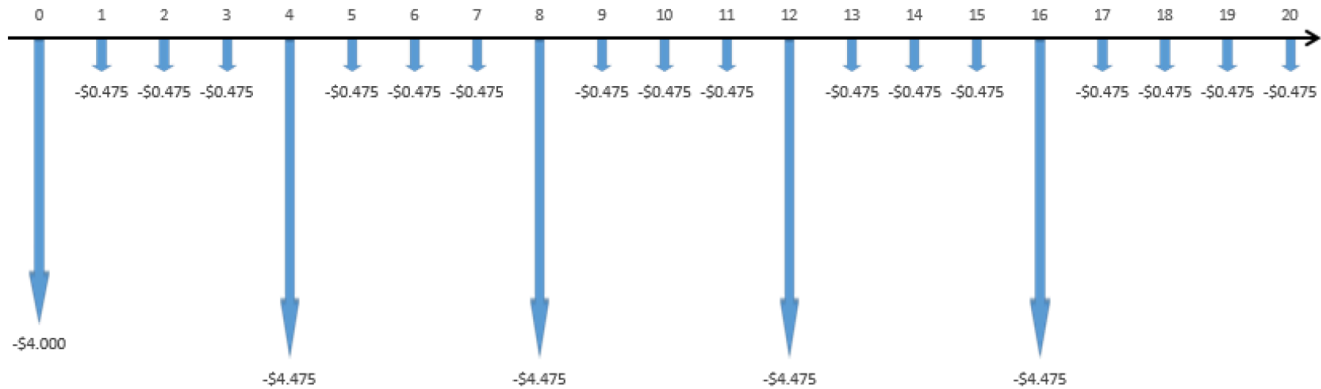
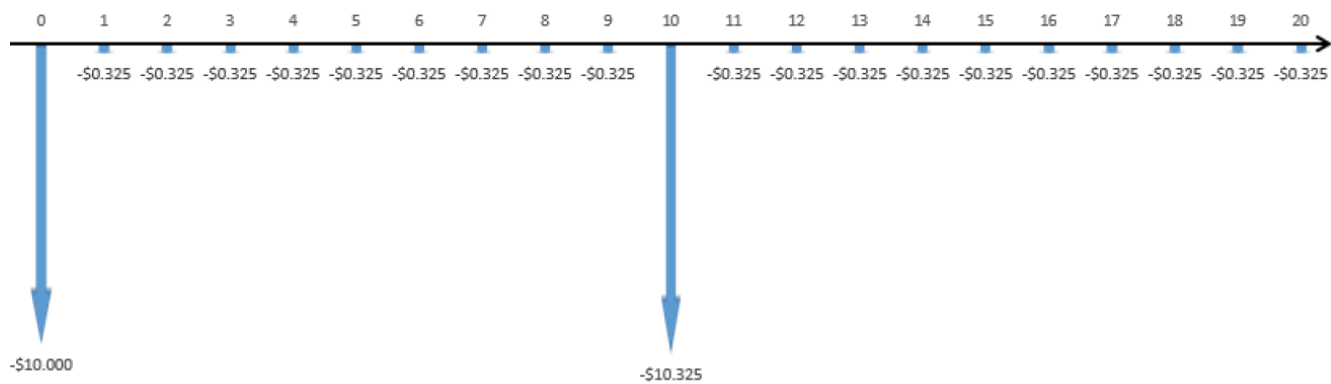
### B3a: Analysis Period = Least Common Multiple of Project Lives

For the LCM method, we set the analysis period equal to the least common multiple of all project lives (in this case, it's 20 years) and compare NPVs.

Step 1: Draw the cash flows for each project over the 20-year analysis period (values shown in millions).

#### Truckasaurus:



**Trashgobbler:****Garbotron****3000:**

Step 2: Calculate the NPV of each alternative (watch those N values!).

$$NPV_{Truck} = -\$5 \text{ million} \left( 1 + \frac{1}{1.05^5} + \frac{1}{1.05^{10}} + \frac{1}{1.05^{15}} \right) - \$375,000 \left[ \frac{1.05^{20} - 1}{0.05(1.05)^{20}} \right]$$

$$NPV_{Truck} = -\$19.07 \text{ million}$$

$$NPV_{Trash} = -\$4 \text{ million} \left( 1 + \frac{1}{1.05^4} + \frac{1}{1.05^8} + \frac{1}{1.05^{12}} + \frac{1}{1.05^{16}} \right) - \$475,000 \left[ \frac{1.05^{20} - 1}{0.05(1.05)^{20}} \right]$$

$$NPV_{Trash} = -\$19.98 \text{ million}$$

$$NPV_{Garbo} = -\$10 \text{ million} \left( 1 + \frac{1}{1.05^{10}} \right) - \$325,000 \left[ \frac{1.05^{20} - 1}{0.05(1.05)^{20}} \right]$$

$$NPV_{Garbo} = -\$20.19 \text{ million}$$

From this method, we can see that the Truckasaurus is the best option.

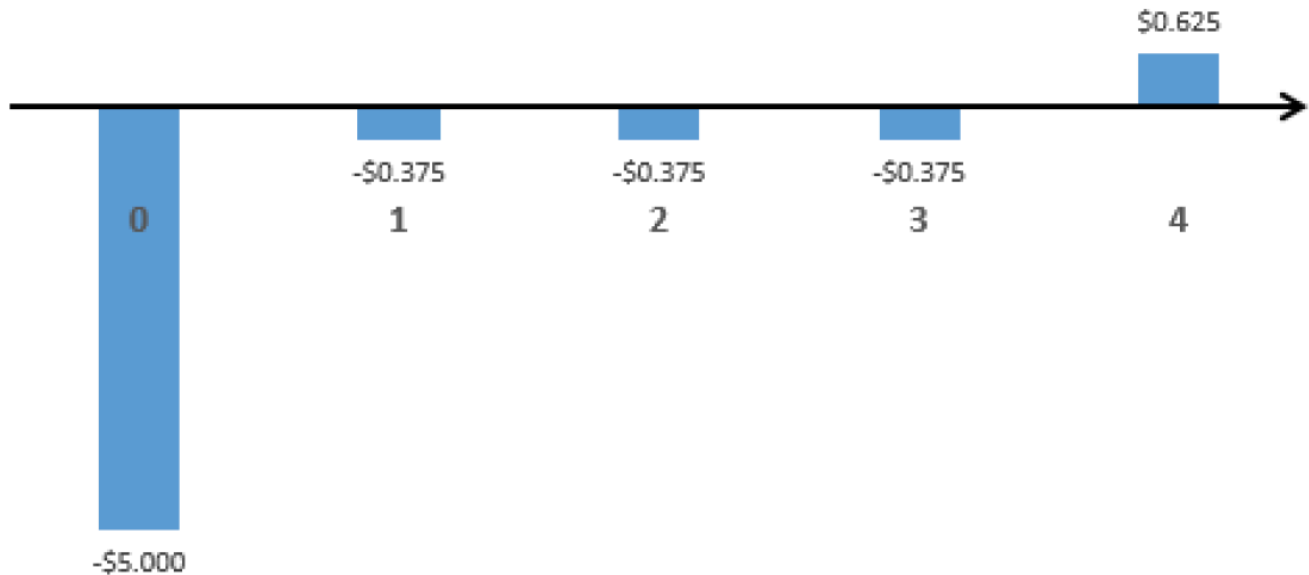
**B3b: Analysis Period = Shortest Project Life**

For this method, we set the analysis period equal to the shortest project life among all the project alternatives

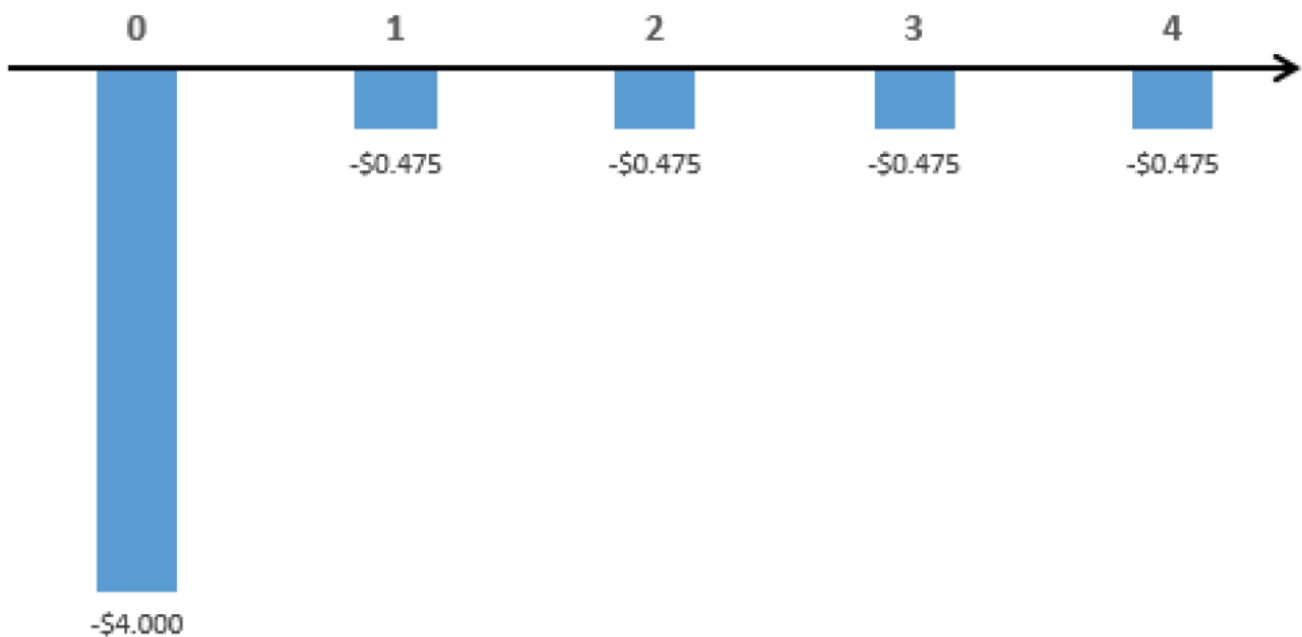
(4 years, in this example). Note that the longer projects will still have some remaining useful life after 4 years. Therefore, we will need to use salvage values at the end of the analysis period. As these were not given, we will estimate these values in Step 2.

Step 1: Draw cash flows.

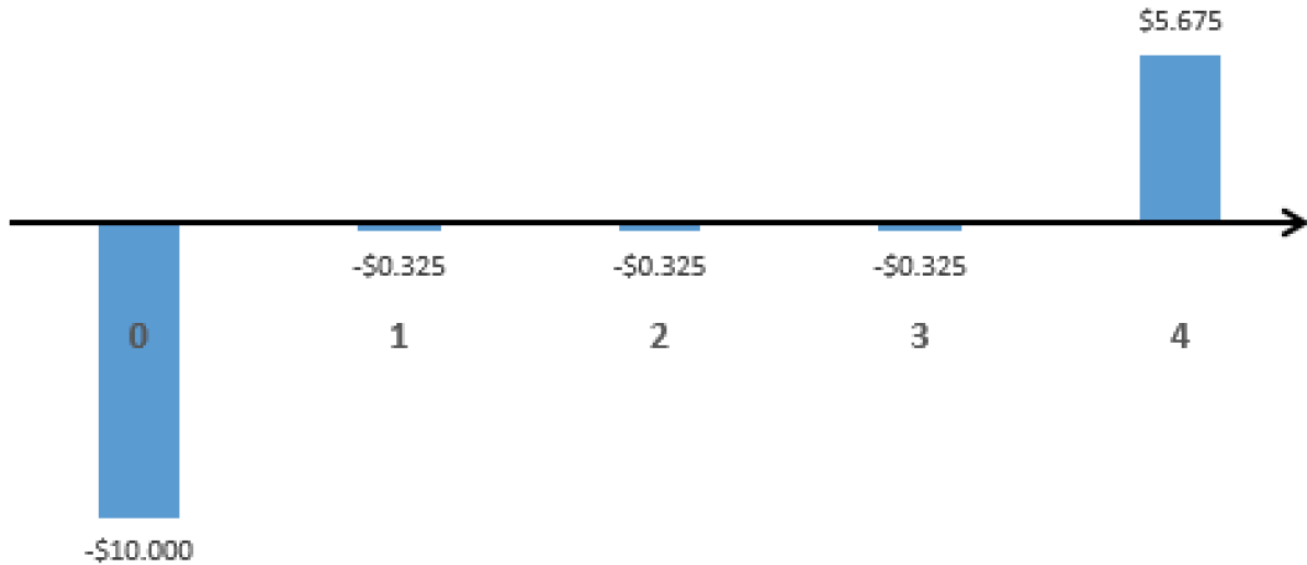
**Truckasaurus:**



**Trashgobbler:**





**Garbotron****3000:**

Step 2: Calculate salvage values for the Truckasaurus and Garbotron 3000.

Salvage values are not given in the problem statement, so we will estimate the salvage values using linear interpolation.

$$S_{Truck} = C_{Truck} \left( \frac{\text{Remaining life}}{\text{Project life}} \right) = \$5 \text{ million} \left( \frac{5 \text{ years} - 4 \text{ years}}{5 \text{ years}} \right) = \$1 \text{ million}$$

$$S_{Garbo} = C_{Garbo} \left( \frac{\text{Remaining life}}{\text{Project life}} \right) = \$10 \text{ million} \left( \frac{10 \text{ years} - 4 \text{ years}}{10 \text{ years}} \right) = \$6 \text{ million}$$

Step 3: Calculate all NPVs to compare projects using the following general equation:

$$NPV = \text{Purchase Price} + \text{Maintenance Costs} + \text{Salvage Value}$$

So we have:

$$NPV_{Truck} = -\$5 \text{ million} - \$375,000 \left[ \frac{1.05^4 - 1}{0.05(1.05)^4} \right] + \frac{\$1 \text{ million}}{1.05^4} = -\$5.51 \text{ million}$$

$$NPV_{Trash} = -\$4 \text{ million} - \$475,000 \left[ \frac{1.05^4 - 1}{0.05(1.05)^4} \right] = -\$5.68 \text{ million}$$

$$NPV_{Garbo} = -\$10 \text{ million} - \$325,000 \left[ \frac{1.05^4 - 1}{0.05(1.05)^4} \right] + \frac{\$6 \text{ million}}{1.05^4} = -\$6.22 \text{ million}$$

Again, we find that the Truckasaurus is the cheapest option.

**B3c: Analysis Period = Longest Project Life**

For this method, the analysis period will be equal to the longest project life among the alternatives, which is 10

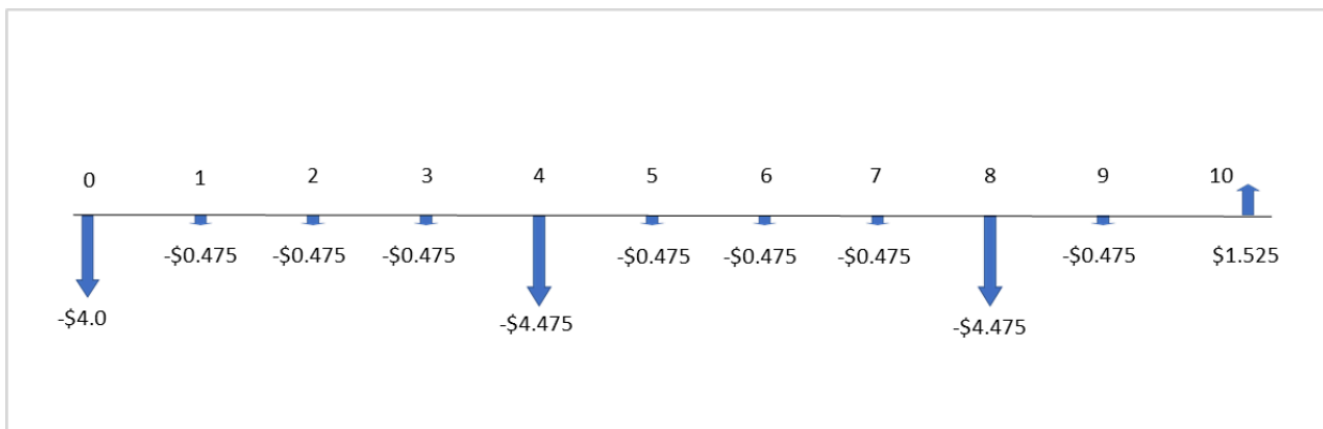
years in this example. The Truckasaurus and Trashgobbler will be replaced by identical copies of themselves for the length of the analysis period. However, the Trashgobbler has a life of 4 years, which does not divide nicely into the 10-year analysis period. This means it will have to be repeated twice, and the third copy will have to be salvaged after the 10th year (its second year of use).

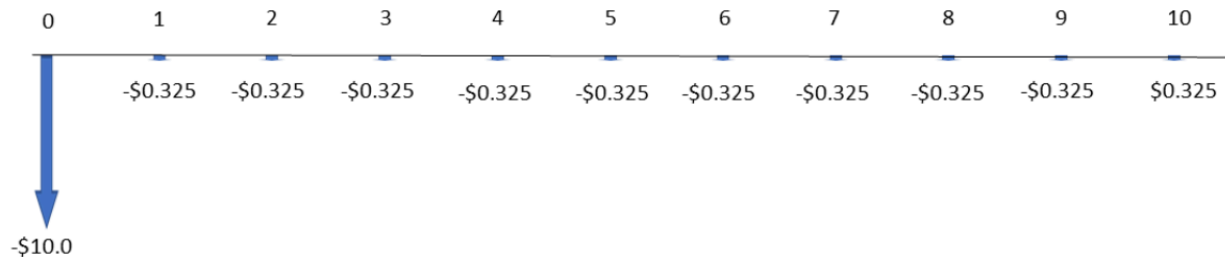
Step 1: Draw the cash flows.

#### Truckasaurus:



#### Trashgobbler:



**Garbotron****3000:**

Step 2: Calculate the salvage value of the Trashgobbler.

$$S_B = C_{Bleft} \left( \frac{\text{Remaining life}}{\text{Project life}} \right) = \$4 \text{ million} \left( \frac{2 \text{ years}}{4 \text{ years}} \right) = \$2 \text{ million}$$

Step 3: Calculate NPVs and compare.

$$NPV_{Truck} = -\$5 \text{ million} \left( 1 + \frac{1}{(1.05)^5} \right) - \$375,000 \left[ \frac{1.05^{10} - 1}{0.05(1.05)^{10}} \right] = -\$11.8 \text{ million}$$

$$NPV_{Trash} = -\$4 \text{ million} \left( 1 + \frac{1}{(1.05)^4} + \frac{1}{(1.05)^8} \right) - \$475,000 \left[ \frac{1.05^{10} - 1}{0.05(1.05)^{10}} \right] + \frac{\$2 \text{ million}}{1.05^{10}} = -\$12.4 \text{ million}$$

$$NPV_{Garbo} = -\$10 \text{ million} - \$325,000 \left[ \frac{1.05^{10} - 1}{0.05(1.05)^{10}} \right] = -\$12.5 \text{ million}$$

As with the other 2 methods, we see that the Truckasaurus is still the cheapest option.

As shown in the example above, in the case of unequal project lives with a finite analysis period, the choice of the analysis period should not affect the resulting decision. There are, however, a few considerations to take into account when choosing an analysis period for this case:

- Are salvage values given? Setting the analysis period equal to the longest or shortest project life requires salvage values, or at least reasonable estimates. If the projects' salvage values are not known and cannot be reasonably estimated, it may be safest to use the LCM method to avoid introducing errors into the calculation.
- Is the analysis period reasonable? For example, if we are comparing three projects with lives of 5, 6, and 7 years, then the lowest common multiple of project lives is 210 years – probably not a reasonable time frame for analysis! Make sure to choose an analysis period that makes sense for the situation at hand.

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## 5.4 Evaluation of Risk

In the examples we have covered so far, we have started out by saying things like “the project will have an annual revenue of \$2000” or “the equipment will have a salvage value of \$500 after 10 years”. But what if these assumptions are wrong? In reality, a project may not be worth what you initially expect, and this discrepancy introduces risk. Using a MARR for discounting cash flows allows us to roughly control for risk (as discussed in Section 5.1); but what about unforeseen circumstances that might create a large impact? For example, what if the economy slows down unexpectedly, reducing sales by 20% of a company’s estimate? What if the company’s borrowing rate changes? Consider the effect on businesses when the price of oil fell dramatically in 2014. Crude oil prices fell from \$111 USD/barrel in June 2014 to \$48 USD/barrel in January 2015, devastating Alberta’s provincial economy.

If the project you are undertaking has very certain outcomes, or will not have severe negative implications if unsuccessful, then you may be able to rely on the basic approaches we have covered already in this chapter. Otherwise, you will want to prepare for risk in your project. In this section, we will see how to analyze and account for risk.

### Terminology

Here are some basic terms that will be used in this section.

- **Risk:** The chance that a project’s actual return will differ from its expected return.
- **Risk analysis:** Identifying and evaluating the impact of various risks on the financial success of a project.
- **Base case:** A scenario representing the most likely case for a project, where the value of all project variables and cash flows is set at an average estimate. We measure risk by comparing different scenarios to the base case.
- **Break-even point:** The value of a specific project variable at which the overall project transitions from unprofitable to profitable (i.e. the project’s NPV is 0)
- **Scenario:** A possible set of conditions for a project. Estimates of project variables can be adjusted to create and compare different scenarios.

## Methods of Evaluating Risk

We will cover three methods for evaluating risk. Below, we list each method and its application. We will review each method in more detail in the following sections.

- Sensitivity analysis → Used for identifying the project variables that, when varied, have the largest effect on the value of the project.
- Scenario analysis → In this method, a project's base case is compared to one or more alternative scenarios (such as the best and worst cases) to identify the most extreme and most likely possible outcomes.
- Break-even analysis → This method identifies the value of a particular project variable that would cause the project to break even.

### 5.4.1 Sensitivity Analysis

In sensitivity analysis, we want to find the variables that have the largest effect on the project's value. For example, if you were opening a commercial bakery whose profits were very sensitive to the price of grain, and grain prices suddenly skyrocketed, your business could be in real trouble. So how do we apply this? Whichever value your decision is most sensitive to, you want to be as certain of as possible. You want to put extra time and effort into getting an accurate estimate. Starting with a reasonable range of values for the variables of your project is an extremely important step. If you are recommending a project to a manager, etc. you want to let them know your assumptions, what the decision relies on (e.g. this project is very sensitive to the price of oil).

There are two main types of sensitivity analysis: univariate analysis and multivariate analysis. In univariate analysis, we alter the value of only one variable at a time, while holding all others constant. In multivariate analysis, we alter the values of multiple variables at one time. In some cases, multivariate analysis can reflect more realistic risk scenarios (e.g. an increase in gasoline prices might cause cost variables for "shipping" and "sales overhead" to increase at the same time, if your salespeople are reimbursed for gas mileage). For the sake of simplicity, this chapter will only cover examples of univariate sensitivity analysis.

Here are the steps to performing univariate sensitivity analysis.

1. Identify and list all variables that factor into the value of the project.
2. Establish a base case of cash flow values and conditions that form a reasonable starting point for analysis. These are generally based on estimates of the most likely values.
3. Analyze the project's cash flows using the base-case values. This method of analysis may be NPV, AEV, etc.
4. **Set a range of possible values for each variable.** This is the range of potential values in which the variable could fall. It is usually expressed as a percentage of the base value (e.g. a salvage value could be \$15,000  $\pm$ 20%) or a constant value (e.g. 10 years  $\pm$ 1 year).
5. Adjust just one variable and recalculate the NPV of the project. Leave all other variables at their base-

case values.

6. Continue to adjust the same variable incrementally within its range, until the entire range is covered, recalculating the project's NPV each time the variable is adjusted. When adjusting the variable, you will likely want to choose some intermediate points between the two extremes – for example, if the interest rate varies between 6% and 14%, you might calculate project NPVs setting the interest rate at 6%, 8%, 10%, 12%, and 14%. Note that in most cases you only need to test the extreme (highest and lowest) values of the range (e.g. 6% and 14%) because the relationship between the variable and the NPV is linear. If the relationship is non-linear, then you will need to calculate intermediate values.
7. **Repeat steps 5 and 6 for all other variables.** Remember, when moving on to each new variable, to reset all other variables to their base-case values.
8. **Record each project NPV and its corresponding variable values.** By examining the data, you can determine which variables have the greatest effect on the project's NPV. We generally look at the extreme values for each variable; the variable which gives an NPV furthest from the base case NPV is the most sensitive.
9. (Optional) Plot the data on a tornado or spider diagram to visually illustrate which variables have the greatest effect.

In the steps above, we defaulted to using NPV. However, note that sensitivity analysis can be used for more than just NPV – for example, it can also be used to calculate AEV. The general process for this is the same.

The key difference between univariate and multivariate analyses is that in Step 5 we may adjust the values of more than one variable at a time.

### Displaying Sensitivity Analysis Results

As mentioned in the sensitivity analysis steps, there are two particularly useful charts for displaying the results of sensitivity analysis: sensitivity diagrams and tornado diagrams. Both can be done using a spreadsheet program. For example, the table of results shown in Figure 1 below could be used to generate the diagrams shown in Figures 2 and 3. You will notice that in this example, not all of the variables vary by the same amount. This is a realistic expectation, since some factors like fixed cost and demand might be more predictable than other variables under certain circumstances.

NPV	-20%	-15%	-10%	-5%	0%	5%	10%	15%	20%
Unit Price	\$(57.00)	\$9,999.10	\$20,055.57	\$30,112.03	\$40,168.50	\$50,225.00	\$60,281.43	\$70,337.90	\$80,394.36
Demand		19,049.92	26,089.45	33,128.97	40,168.50	47,208.03	54,247.55	61,287.08	
Variable Cost	52,236.26	49,219.32	46,202.38	43,185.44	40,168.50	37,151.56	34,134.62	31,117.68	28,100.74
Fixed Cost			42,179.79	41,174.15	40,168.50	39,162.85	38,157.21		
Salvage	37,782.05	38,378.66	38,975.28	39,571.89	40,168.50	40,765.11	41,361.72	41,958.34	42,554.95

Figure 1: Sensitivity Analysis Results

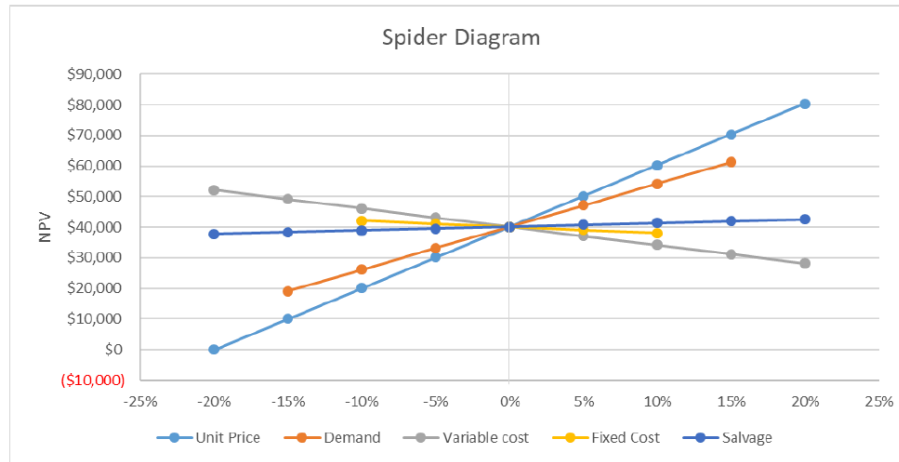


Figure 2: Spider Diagram of the sensitivity analysis results

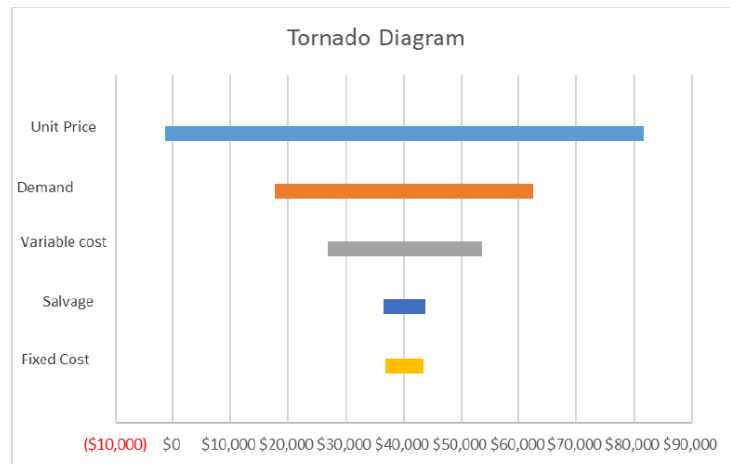


Figure 3: Tornado Diagram of the sensitivity analysis results

Sensitivity diagrams and tornado diagrams both illustrate the same concept: they show how the project's NPV (or AEV, etc.) changes as the variables change. Each line shows how the project value changes as one variable is adjusted, keeping all other variables constant. The number of lines will be equal to the number of variables you are analyzing. The only thing that differs between the two diagrams is how they are presented.

In the sensitivity diagram, we plot project value on the y-axis, and the percent change of the variable along the x-axis. The point at which all lines intersect is the base-case value of the project. Once the data are plotted for each variable, we can see how each variable affects the project value (i.e. whether an increase in the variable causes an increase or decrease in project value, and by how much). The line with the steepest slope will generate the most extreme NPVs, and will reflect the variable which most significantly affects the value of the project. Similarly, a

line with a mild slope will produce less extreme NPVs, and will reflect a less sensitive variable. Looking at the figure above, Unit Price is the most sensitive variable (steepest slope), and Salvage is the least sensitive variable.

On a tornado diagram, the range of values for each variable is lines is plotted as a horizontal line, across the project values on the x-axis. On a tornado diagram, the longer a line is, the more the variable affects the project's value. The lines are typically arranged by length, with the longest line at the top, so that the diagram resembles a tornado. As Figure \_\_\_\_ shows, Unit Price has the longest line, and is the most sensitive variable. The base-case NPV is noted by the vertical line on the diagram, which is in the exact centre of each horizontal line.

### 5.4.2 Scenario Analysis

Scenario analysis is an adapted form of sensitivity analysis. In scenario analysis, we compare scenarios – essentially sets of values for each variable in the project. The scenarios are based on the likely range of values for each variable, as well as the probability of each set of values. While it may be useful to check several combinations of variables, it should be noted that not all scenarios are feasible or likely, and so analyzing every possible scenario is unnecessary. For example, you would likely not consider a scenario where demand for your product is extremely low but your monthly shipping costs are extremely high, since your shipping costs should be somewhat dependent on the amount of product you are selling.

Scenario analysis allows you to prepare for the outcomes of multiple likely situations, and understand the most extreme impacts of the risks facing your project. It can also be a useful tool to help decision makers understand the risks and benefits of a project in a way that makes intuitive sense.

Here are the steps to performing a scenario analysis.

1. Identify and list all variables that factor into the value of the project.
2. Establish a base case scenario – a set of cash flow values and conditions that form a reasonable starting point for analysis. These are typically the most likely estimates for each variable.
3. Calculate the NPV (or AEV, etc.) of the project for the base case scenario.
4. Establish a new scenario. For each scenario being analyzed, obtain (or estimate) the values of all variables. Most often, we use a best-case and a worst-case scenario.
5. Use the scenario from Step 4 to calculate the NPV of the project.
6. Repeat steps 4 and 5 for all relevant scenarios.
7. Compare the NPV of all scenarios, including the base case scenario.

As with sensitivity analysis, it is not necessary to use NPV as the measure of a project; AEV or another analysis method can also be substituted.

The goal of scenario analysis is to identify the most extreme (best and worst case) and most likely project outcomes. It might not be sufficient to only consider the best and worst case, especially if the best and worst



case are both highly unlikely. It may be most useful to add more (and, hopefully, highly probable) scenarios by considering other known conditions surrounding the project.

### 5.4.3 Break-Even Analysis

Sometimes we are interested in how much our costs would have to increase, or how much our revenues would have to decrease, before a project begins to lose money. In these cases, we can use break-even analysis. Break-even analysis tells us what value of a particular variable will cause the project to exactly break even. Imagine that you run a bicycle repair shop, and you discover that your business will only break even if you charge at least \$60 per repair. Or that at your current prices, you could increase your employees' wages to \$21/hour before your business began to lose money. These pieces of information could be very valuable to you when making decisions about pricing and employee salaries.

Break-even analysis is another valuable tool for informing decisions about businesses and projects, especially when considering a new project to invest in. Understanding the changes to certain variables that will make a project profitable gives a good basic indication of whether it is worth investment.

Here are the steps to performing break-even analysis:

1. Identify and list all variables that factor into the value of the project.
2. Establish a base case – a set of cash flow values and conditions that form a reasonable starting point for analysis.
3. Determine a formula for the project's NPV.
4. Set the NPV equal to zero, and solve for a chosen "test" variable.
5. Calculate the value of the test variable\*. This will be the break-even point for the test variable. Make sure all other variables are held at their base-case values, and the NPV is zero.
6. Repeat steps 4 and 5 for each variable. Each time you choose a new variable, remember to reset all other variables' values to their base-case values.

**\*Note about step 5:** Usually, the variable can be solved for directly. However, if the variable in question appears in multiple places in the equation, and can't be isolated (this is common for the discount rate), special methods will have to be used. If this is the case, you may have to perform iteration. Excel is particularly helpful in this regard. You may have to look up fixed-point iteration or another, more advanced method to iterate in Excel.

Break-even analysis allows us to see how much we can allow each variable to change before we will refuse the project. If the break-even point falls within the estimated range of any of the variables, the project should probably be rejected. For example, if we expect unit costs to be somewhere between \$18 and \$22, and the project breaks even with unit costs of \$21, the project may be too risky to undertake.

### 5.5.1 Example of Sensitivity, Scenario and Break-Even Analysis

#### Sensitivity Analysis: 1-year

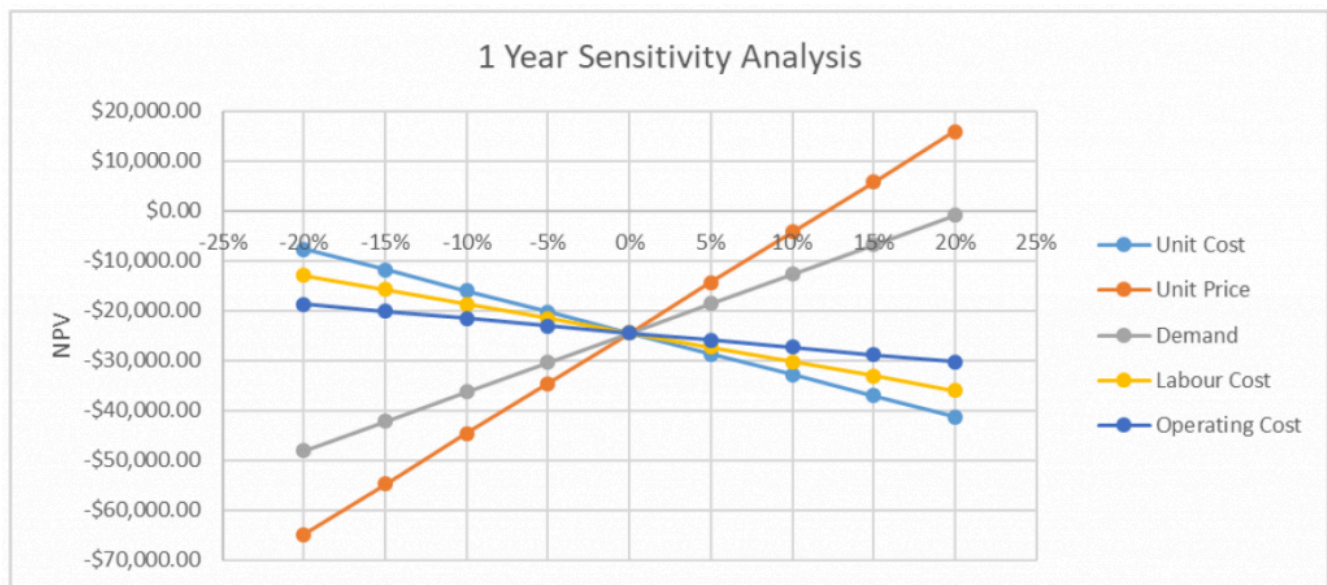
To begin the sensitivity analysis, create a table of cash flows for the project, calculating the NPV of the project over its duration. Begin by using the base case values supplied for all variables. Your table could look something like this:

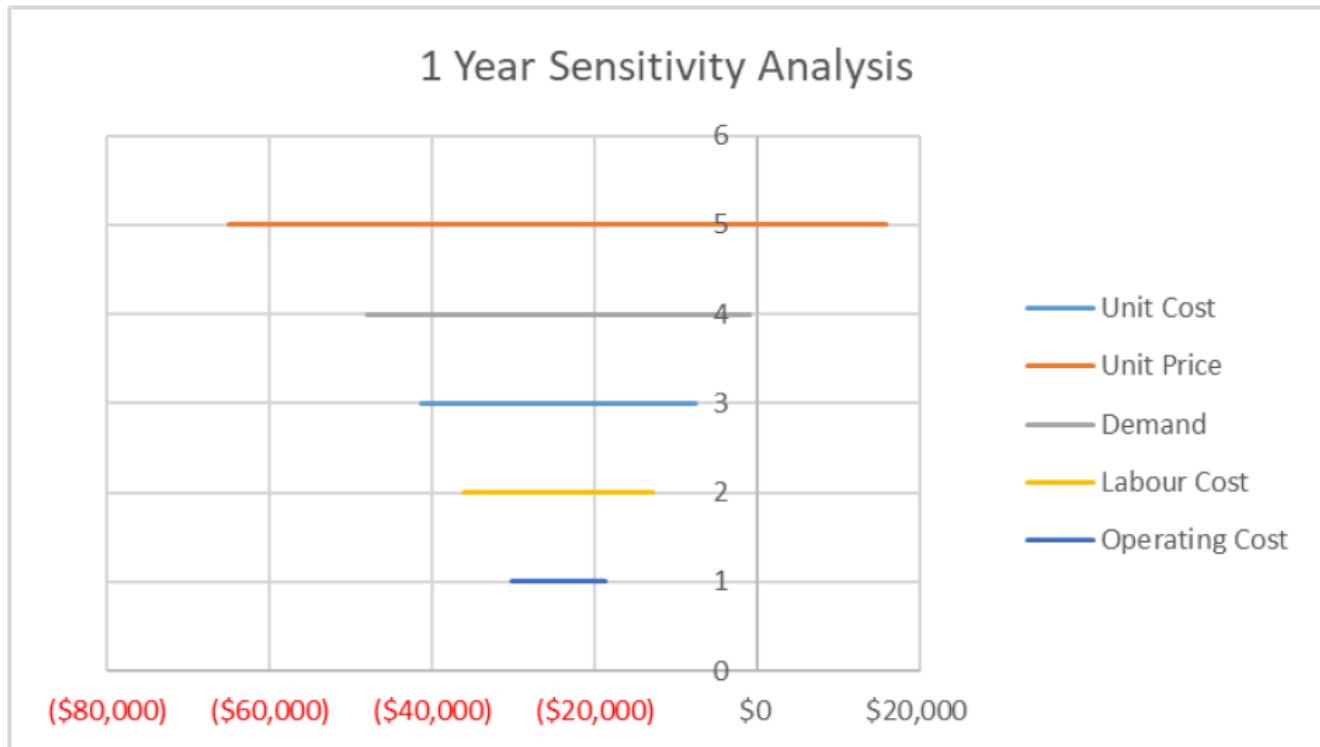
Year	0	1
<b>Revenues:</b>		
Unit Price		\$6.00
Demand		35,000
Sales Revenue		\$210,000.00
Salvage Value		\$150,000.00
<b>Expenses:</b>		
Investment	\$200,000.00	
Unit Cost		\$2.50
Demand		35,000
Sales Cost		\$87,500.00
Labour Cost		\$60,000.00
Operating Cost		\$30,000.00
<b>Net Profit</b>	-\$200,000.00	\$182,503.50
<b>Net Present Value</b>	-\$200,000.00	\$175,484.13
<b>Total NPV:</b>		-\$24,515.87

The total NPW you calculate with this table represents a single data point for this analysis. To fill in the additional data points, you must alter the value of a single variable (e.g. Unit Cost) at a time, and find how a certain change (e.g. +5%) impacts the total NPV. For the purposes of this example, we will evaluate all of our variables in the range of  $\pm 20\%$ , with increments of 5%. After evaluating every scenario for every variable, you should arrive at a table of results like this:

	-20%	-15%	-10%	-5%	0%	5%	10%	15%	20%
Unit Cost	-\$7,688.46	-\$11,895.31	-\$16,102.16	-\$20,309.01	-\$24,515.87	-\$28,722.72	-\$32,929.57	-\$37,136.42	-\$41,343.27
Unit Price	-\$64,901.63	-\$54,805.19	-\$44,708.75	-\$34,612.31	-\$24,515.87	-\$14,419.42	-\$4,322.98	\$5,773.46	\$15,869.90
Demand	-\$48,073.56	-\$42,184.13	-\$36,294.71	-\$30,405.29	-\$24,515.87	-\$18,626.44	-\$12,737.02	-\$6,847.60	-\$958.17
Labour Cost	-\$12,977.40	-\$15,862.02	-\$18,746.63	-\$21,631.25	-\$24,515.87	-\$27,400.48	-\$30,285.10	-\$33,169.71	-\$36,054.33
Operating Cost	-\$18,746.63	-\$20,188.94	-\$21,631.25	-\$23,073.56	-\$24,515.87	-\$25,958.17	-\$27,400.48	-\$28,842.79	-\$30,285.10

We can display the results of a sensitivity analysis graphically using either a sensitivity diagram or a tornado plot. This can be accomplished in a spreadsheet program by isolating the important results from each variable as inputs for a graph:



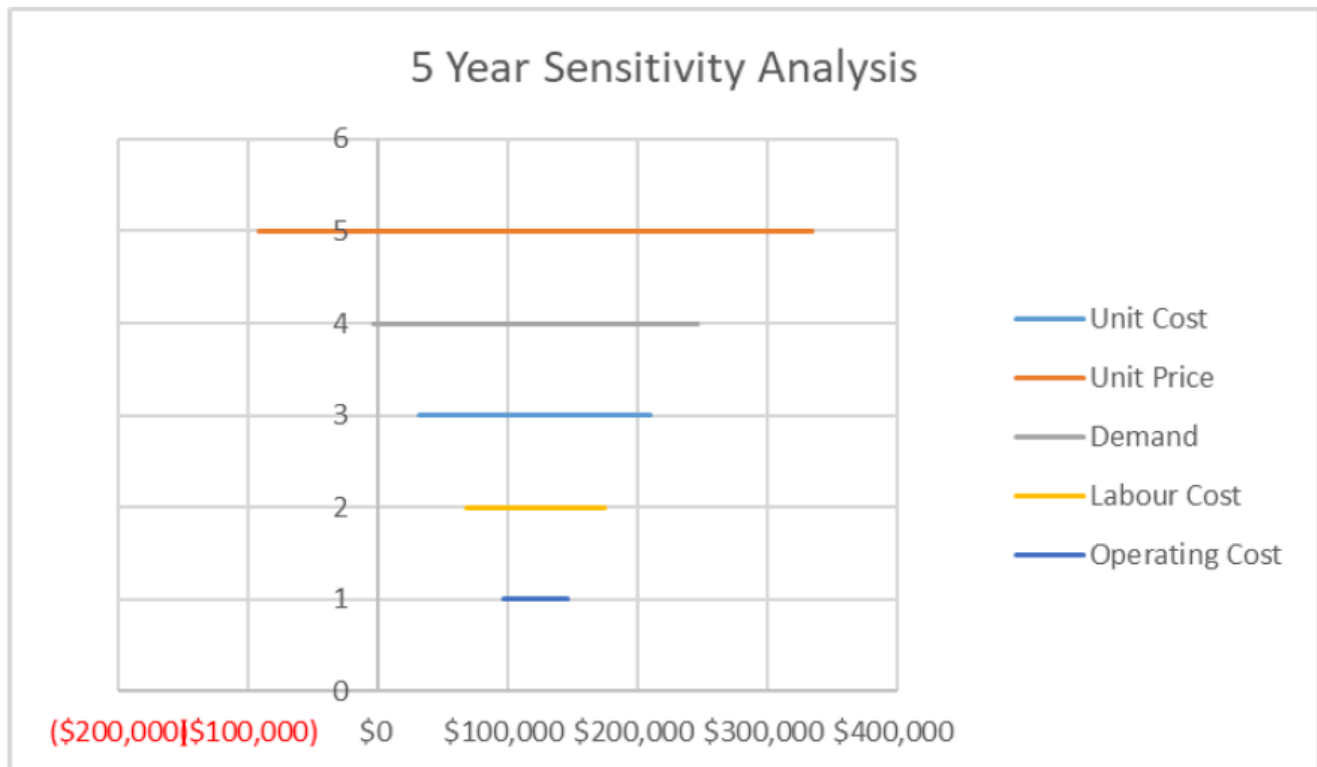
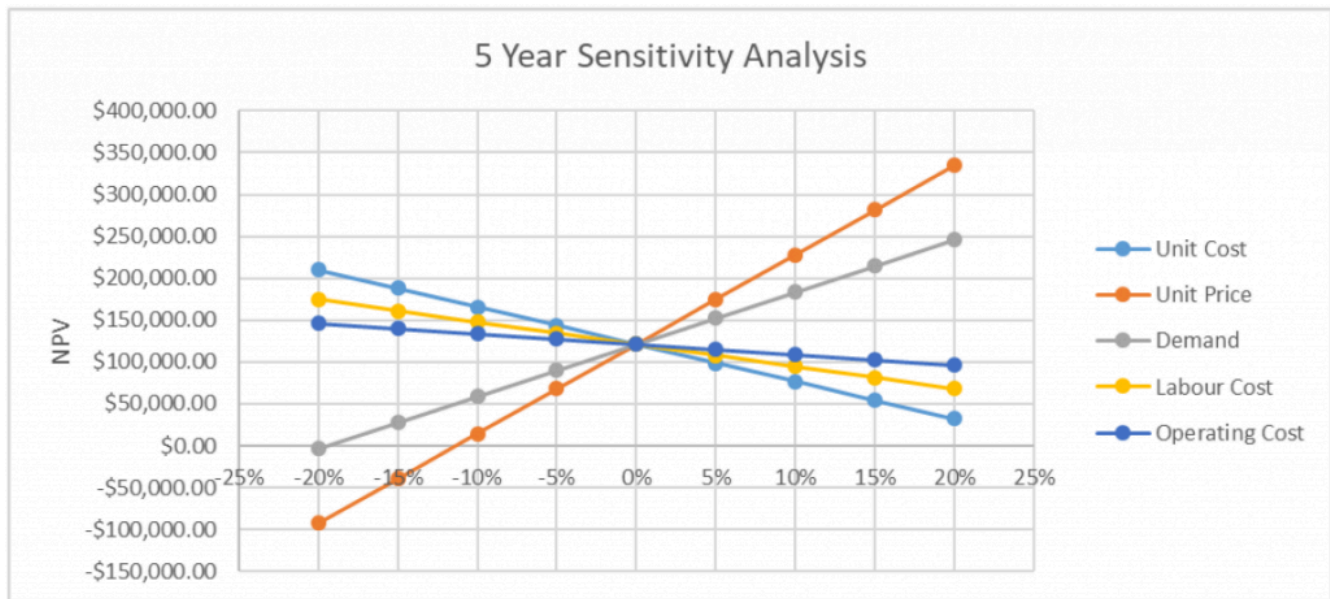


#### Sensitivity Analysis: 5 year

If we follow the same methodology for our 5-year estimates, we should produce the following results and graphs.

Year	0	1	2	3	4	5
<b>Revenues:</b>						
Unit Price		\$6.00	\$6.00	\$6.00	\$6.00	\$6.00
Demand		40,000	40,000	40,000	40,000	40,000
Sales Revenue		\$240,000.00	\$240,000.00	\$240,000.00	\$240,000.00	\$240,000.00
Salvage Value						\$120,000
<b>Expenses:</b>						
Investment	\$200,000.00					
Unit Cost		\$2.50	\$2.50	\$2.50	\$2.50	\$2.50
Demand		40,000	40,000	40,000	40,000	40,000
Sales Cost		\$100,000.00	\$100,000.00	\$100,000.00	\$100,000.00	\$100,000.00
Labour Cost		\$60,000.00	\$60,000.00	\$60,000.00	\$60,000.00	\$60,000.00
Operating Cost		\$30,000.00	\$30,000.00	\$30,000.00	\$30,000.00	\$30,000.00
<b>Net Profit</b>	-\$200,000.00	\$50,003.50	\$50,003.50	\$50,003.50	\$50,003.50	\$170,003.50
<b>Net Present Value</b>	-\$200,000.00	\$48,080.29	\$46,231.05	\$44,452.93	\$42,743.20	\$139,730.48
<b>Total NPV: \$121,237.95</b>						

	-20%	-15%	-10%	-5%	0%	5%	10%	15%	20%
Unit Cost	\$210,276.62	\$188,016.96	\$165,757.29	\$143,497.62	\$121,237.95	\$98,978.28	\$76,718.61	\$54,458.95	\$32,199.28
Unit Price	-\$92,454.86	-\$39,031.66	\$14,391.54	\$67,814.75	\$121,237.95	\$174,661.15	\$228,084.36	\$281,507.56	\$334,930.76
Demand	-\$3,413.07	\$27,749.68	\$58,912.44	\$90,075.19	\$121,237.95	\$152,400.71	\$183,563.46	\$214,726.22	\$245,888.97
Labour Cost	\$174,659.82	\$161,304.35	\$147,948.88	\$134,593.42	\$121,237.95	\$107,882.48	\$94,527.02	\$81,171.55	\$67,816.08
Operating Cost	\$146,018.07	\$139,823.04	\$133,628.01	\$127,432.98	\$121,237.95	\$115,042.92	\$108,847.89	\$102,652.86	\$96,457.83



### Scenario Analysis

To conduct a scenario analysis, we return to the same table of cash flows we used to compute our sensitivity analysis, and reference the either all of the provided best case values, or all of the provided worst case values, in order calculate a total NPV for the project. We will also include the base case calculation (completed during sensitivity analysis) for reference. This process should produce the following set of NPV values:

	1-Year	5-Year
Best Case:	\$126,928.85	\$794,589.43
Worst Case:	-\$137,499.04	-\$552,113.53
Base Case:	-\$24,515.87	\$121,237.95

### Break-Even Analysis

To begin break-even analysis, we return our entire table of cash flows to its base case values. Then, for a single variable (e.g. Unit Cost), we determine what value it would need to hold for the total NPV of the project to equal zero (i.e. the break-even point), with all other base case values remaining constant. We could accomplish this by writing out a long equation for the NPV, setting it equal to zero, and then solving for our isolated variable. However, this process would be very time intensive, and if you are using a spreadsheet computer program, there will analysis methods you can run to quickly solve for the value of each variable.

This process should produce the following set of breakeven points:

Break-even Point	1-Year	5-Year
Unit Cost	\$1.77	\$3.18
Unit Proce	\$6.73	\$5.32
Demand	42,285	32,219
Labour Cost	\$34,503.50	\$87,233.33
Operating Cost	\$4,503.50	\$57,233.33

### Conclusions

From the sensitivity analysis, we can conclude that unit price and demand are the two most sensitive variables, and also the only two variables that over 5-years have the potential to generate a negative NPV for the project. From this, we can conclude that setting a high enough price for the hamburgers is very important to the profitability of the business. However, we should also consider the real-world relationship between the price of the burgers and the demand for them; finding a suitably high price that will not hinder demand for your product will be one of the most important considerations for this business.

We can also see that the majority of values generated in this analysis, including the base case, for the 1-year estimates are negative, while for the 5-year estimates the majority are positive. This is largely due to the salvage value of the food trucks falling by \$50,000 in the first year, and thus making it difficult to generate a net profit on the business with only one year of sales. We can conclude that this business is a safer and more lucrative investment over 5-years, and likely not even worth investment on a 1-year basis.

From the scenario analysis, we can conclude that there is a wide range of possible outcomes for this business, ranging from extremely profitable (almost quadrupling the initial investment in 5 years) to extremely unprofitable (losing on net around \$500,000). While our sensitivity analysis showed us that this is likely to be a fairly safe investment, the potential for multiple negative variables to add up means that this project could lose a significant amount of money in a worst-case scenario. A 1-year investment involves less catastrophic risk, but because its base case still gives a negative NPV it would not be a wise investment. If you were investing your own savings in this business, you might consider the potential risk for a 5-year term to be too high to consider investing.

Finally, the break-even analysis provides us with useful values for the operation of the business. For example, if after opening the business, the unit cost of your burgers were to rise above its break-even point, you would know that you were at risk of having a negative project NPV. You could then adjust your business practices to correct for this, by increasing burger prices, running advertising to increase sales, reducing operating hours to limit labour costs, or seeking ways to operate the food truck equipment on a lower budget.

If we were attempting to make the business profitable on a 1-year basis, we can see that adjusting some variables would be more realistic than attempting to adjust others. Finding cheaper ingredients to reduce hamburger costs to \$1.77/unit, or raising prices to \$6.73/burger seems reasonable, but reducing operating costs to below \$4,503 (an 85% reduction) would be nearly impossible.



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# Chapter 6: Evaluations

## Introduction

Imagine that you are a senior engineer working for the provincial government. You are assigned to lead a group assessing the feasibility of a proposed nuclear power station just north of Prince Albert. The power station will operate for several decades, requiring long-term estimates for maintenance and operational costs, as well as projected revenues. Daily operations at the station will create jobs for workers from nearby communities, and draw qualified professionals from across the country. Effects on the surrounding environment must be considered, including the safe storage of waste materials and the impact of decommissioning the station at the end of its life cycle. The station will also reduce provincial CO<sub>2</sub> emissions by replacing fossil fuel energy sources that are currently used. Aside from power generation, the nuclear reactor at the site could be used to produce important medical isotopes which local hospitals would otherwise be required to import from international suppliers.

What methods should your group use when preparing the recommendation for this project? You are now familiar with many tools for evaluating projects; these tools are a dependable resource for evaluating investments that involve a straightforward set of cash flows. However, when considering a complex, long-term project where the potential costs and benefits are not solely monetary, more factors must also be considered.

Some questions your group of engineers might consider are:

- What is the total cost of this power station over its full life cycle, including design, construction, operation, and decommissioning?
- Who are the people involved with or affected by (the stakeholders) this project, and how important are their concerns?
- What value should be assigned to the environmental, social, and economic impacts of this project? Can this value be quantified? What is the relative importance of each impact?
- In purely financial terms, is there a benefit to replacing existing fossil fuel power stations with this nuclear power station?
- Can the benefits and costs of the power station be compared to one another to reach a decision?

In this chapter we will discuss the evaluation of complex projects, and how various analysis techniques can be

used in situations like the one given above. The first topics covered will be replacement analysis and life cycle costs, which deal with the broad financial scope of a project. Then we will cover benefit-cost (or cost-benefit) analyses and feasibility studies, and discuss the differences between service projects and revenue projects.

**Key Concepts**

- Replacement Analysis
- Life Cycle Cost
- Benefit-Cost Analysis
- Feasibility Studies
- Services Projects vs Revenue Projects

**Learning Objectives**

After completing this chapter, students should be able to:

- Replacement Analysis
- Life Cycle Cost
- Services Projects vs Revenue Projects
- Benefit-Cost Analysis
- Feasibility Studies

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## 6.1 Replacement Analysis

Any project or investment is bound to have a finite lifespan, after which it will require replacement. Often, the costs of a project will increase over time, which in many cases means the profitability decreases, and eventually the revenue or other benefits gained will be less than the expected profit of replacing it with a new investment. For example, it might be possible to keep your old refrigerator running for many years. However, the cost savings of replacing it with a newer, more energy-efficient model might eventually outpace the cost of buying that new fridge. **Replacement analysis** is used to determine when the cost of replacing a project is preferable to maintaining it.

The replacement does not need to be an identical project or investment. For instance, the original business model for Netflix was a DVD rental company which sent out movies to its customers by mail. As media trends changed, Netflix gradually replaced this model with a service to stream movies and television shows over the internet, and later began to produce their own original content. These decisions were based on an analysis of whether the replacements (the new services) would generate more long term revenue than the company's existing investments.

Replacement analysis is closely related to the concept of **opportunity costs** discussed in Chapter 5: when your money is tied up in a project with a lower rate of return than another available project, you encounter the opportunity cost of not being able to pursue the more lucrative project. Replacement analysis allows you to determine when you should reinvest in a new project.

### 6.1.1 Economic Life vs. Physical Life

In Chapter 5, we introduced the *linear depreciation* method, which allowed us to estimate the salvage value of a project based on its original cost and the amount of time it had been in use. The assumption we made was that a project's value is initially equal to its investment cost, and its value declines steadily over time. In this scenario, the amount of time before the project can no longer be used or its value reaches zero is referred to as its **physical life**. The physical life of a project is the amount of time it can operate while still generating economic value. For example, the physical life of a car is the number of years it can be run before breaking down irreparably.

As discussed in our introduction to replacement analysis, there are situations where we would rather replace a project before the end of its physical life. To build on previous example, the physical life of a car may be 25 years, but after 10 years of ownership we might choose to replace the car with a new vehicle that consumes less fuel and does not require expensive repairs (or just looks prettier). Let's look purely at the economics: if after

10 years, these costs have finally surpassed the price of trading in the car for a new vehicle, then we say that the car has reached the end of its **economic life**. The economic life of an project is the period of time when it is generating greater value than any possible alternatives. A project's economic life is always equal to or shorter than its physical life.

To generate the most economic value, we want to replace our projects at the end of their *economic life*, rather than at the end of their *physical life*. Identifying this optimum replacement point is the basic concept of replacement analysis.

### Example #1

*Sophie runs a printing company which produces books for large publishing houses. The facility she operates uses a set of old printing presses, and she expects that her business might become more profitable if she were to upgrade to newer, more reliable presses that would require less maintenance. Assume that her current set of presses are now valued at \$400,000, and will depreciate linearly (to zero) until the end of their useful life in 25 years. In the next year, she predicts the presses will incur \$40,000 in maintenance costs, and that this amount will increase by 10% each year as the presses age. She could upgrade right now to a new set of presses for \$2,000,000; due to technological advances and more widespread use, over the next 10 years, the price of buying these new presses will decrease linearly to \$1,000,000, and then remain at that price for the next 15 years. The new presses will require negligible maintenance, and the purchase price covers the long-term maintenance. The old set of presses earns \$120,000 per year, and the new set would earn \$300,000 per year. How long should Sophie wait before upgrading her presses? Use a MARR of 4%.*

Step 1: The most useful way to visualize this problem is to create a table which finds the NPV of selling the presses each year. To do this, we consider 4 elements:

1. the salvage value of the old presses,
2. the expense of the new presses,
3. the revenue from the old presses until they are sold, and
4. the revenue from the new presses after they are bought.

We can find (1) by interpolating between \$400,000 and \$0 over 25 years for the old presses, and (2) by interpolating between \$2,000,000 and \$1,000,000 for the new presses over the same time period, as shown below in columns two and three. In the fourth column we use our basic present value formula to find the present value of the sum of the two previous columns. This should yield the following results:

Year	Salvage Value for Old Presses	Purchase Price for New Presses	Total Cost for Replacing Presses	NPV of Cost of Replacing Presses
0	\$400,000	-\$2,000,000	-\$1,600,000	-\$1,600,000
1	\$384,000	-\$1,900,000	-\$1,516,000	-\$1,457,692
2	\$368,000	-\$1,800,000	-\$1,432,000	-\$1,323,964
3	\$352,000	-\$1,700,000	-\$1,348,000	-\$1,198,367
4	\$336,000	-\$1,600,000	-\$1,264,000	-\$1,080,472
5	\$320,000	-\$1,500,000	-\$1,180,000	-\$969,874
6	\$304,000	-\$1,400,000	-\$1,096,000	-\$866,185
7	\$288,000	-\$1,300,000	-\$1,012,000	-\$769,037
8	\$272,000	-\$1,200,000	-\$928,000	-\$678,081
9	\$256,000	-\$1,100,000	-\$844,000	-\$592,983
10	\$240,000	-\$1,000,000	-\$760,000	-\$513,429
11	\$224,000	-\$1,000,000	-\$776,000	-\$504,075
12	\$208,000	-\$1,000,000	-\$792,000	-\$494,681
13	\$192,000	-\$1,000,000	-\$808,000	-\$485,264
14	\$176,000	-\$1,000,000	-\$824,000	-\$475,839
15	\$160,000	-\$1,000,000	-\$840,000	-\$466,422
16	\$144,000	-\$1,000,000	-\$856,000	-\$457,025
17	\$128,000	-\$1,000,000	-\$872,000	-\$447,661
18	\$112,000	-\$1,000,000	-\$888,000	-\$438,342
19	\$96,000	-\$1,000,000	-\$904,000	-\$429,077
20	\$80,000	-\$1,000,000	-\$920,000	-\$419,876
21	\$64,000	-\$1,000,000	-\$936,000	-\$410,748
22	\$48,000	-\$1,000,000	-\$952,000	-\$401,702
23	\$32,000	-\$1,000,000	-\$968,000	-\$392,743
24	\$16,000	-\$1,000,000	-\$984,000	-\$383,880
25	\$0	-\$1,000,000	-\$1,000,000	-\$375,117

Table 6.1: Salvage values and expenses for replacing old printing presses

Step 2: Next, find the NPV of the elapsed maintenance costs each year using the geometric gradient series formula, and add this to the table:

$$NPV_{Maintenance} = \frac{A}{i-g} \left[ 1 - \left( \frac{1+g}{1+i} \right)^N \right] = \frac{\$1500}{0.04-0.1} \left[ 1 - \left( \frac{1+0.1}{1+0.04} \right)^N \right]$$

Step 3: Next, find the NPV of the elapsed revenue from the old presses each year using the equation for a linear series:

$$NPV_{OldRev} = A \left[ \frac{(1+i)^N - 1}{i(1+i)^N} \right] = \$120,000 \left[ \frac{(1+0.04)^N - 1}{0.04(1+0.04)^N} \right]$$

Step 4: Finally, find the NPV of future revenue to be gained from the new presses in future years, by subtracting the value up to the current year from the value of a 25-year series:

$$NPV_{NewRev} = \$300,000 \left[ \frac{(1+0.04)^{25} - 1}{0.04(1+0.04)^{25}} \right] - \$300,000 \left[ \frac{(1+0.04)^N - 1}{0.04(1+0.04)^N} \right]$$

We can add all of these values to the table, and sum all of our NPVs to get a total NPV representing the value of switching to new presses during each year:

Year	NPV of Cost of Replacing Presses	NPV of Maintenance Costs for Old Presses	NPV of Revenue from Old Presses up to Present Year	NPV of Revenue from New Presses in Future Years	Total NPV	Change in NPV
0	-\$1,600,000	\$0	\$0	\$3,124,415.99	\$1,524,416	\$26,923
1	-\$1,457,692	-\$38,462	\$115,385	\$2,932,108.30	\$1,551,339	\$19,083
2	-\$1,323,964	-\$79,142	\$226,331	\$2,747,197.05	\$1,570,422	\$11,450
3	-\$1,198,367	-\$122,169	\$333,011	\$2,569,397.78	\$1,581,872	\$4,000
4	-\$1,080,472	-\$167,679	\$435,587	\$2,398,436.94	\$1,585,873	-\$3,291
5	-\$969,874	-\$215,815	\$534,219	\$2,234,051.52	\$1,582,582	-\$10,448
6	-\$866,185	-\$266,727	\$629,056	\$2,075,988.62	\$1,572,133	-\$17,495
7	-\$769,037	-\$320,577	\$720,247	\$1,924,005.05	\$1,554,638	-\$24,455
8	-\$678,081	-\$377,533	\$807,929	\$1,777,867.01	\$1,530,183	-\$31,352
9	-\$592,983	-\$437,775	\$892,240	\$1,637,349.67	\$1,498,831	-\$38,208
10	-\$513,429	-\$501,493	\$973,307	\$1,502,236.83	\$1,460,623	-\$110,006
11	-\$504,075	-\$568,887	\$1,051,257	\$1,372,320.65	\$1,350,616	-\$111,856
12	-\$494,681	-\$640,169	\$1,126,209	\$1,247,401.24	\$1,238,760	-\$114,023
13	-\$485,264	-\$715,563	\$1,198,278	\$1,127,286.42	\$1,124,737	-\$116,518
14	-\$475,839	-\$795,307	\$1,267,575	\$1,011,791.40	\$1,008,220	-\$119,349
15	-\$466,422	-\$879,652	\$1,334,206	\$900,738.50	\$888,871	-\$122,527
16	-\$457,025	-\$968,862	\$1,398,275	\$793,956.87	\$766,345	-\$126,063
17	-\$447,661	-\$1,063,220	\$1,459,880	\$691,282.22	\$640,281	-\$129,972
18	-\$438,342	-\$1,163,021	\$1,519,116	\$592,556.59	\$510,309	-\$134,265
19	-\$429,077	-\$1,268,580	\$1,576,073	\$497,628.11	\$376,044	-\$138,959
20	-\$419,876	-\$1,380,229	\$1,630,839	\$406,350.72	\$237,085	-\$144,069
21	-\$410,748	-\$1,498,319	\$1,683,499	\$318,584.00	\$93,016	-\$149,613
22	-\$401,702	-\$1,623,222	\$1,734,134	\$234,192.92	-\$56,597	-\$155,609
23	-\$392,743	-\$1,755,331	\$1,782,821	\$153,047.66	-\$212,205	-\$162,077
24	-\$383,880	-\$1,895,061	\$1,829,636	\$75,023.36	-\$374,282	-\$169,039
25	-\$375,117	-\$2,042,853	\$1,874,650	\$0.00	-\$543,321	

Table 6.2 Total net present value of the business each year, if the old printing presses were replaced in that year.

All of the NPV totals in the final column are positive, so we can conclude that switching to new presses would be a lucrative move at any point in time. However, we can also see that the peak NPV value is at Year 6, with a maximum NPV of \$1,544,345. **If Sophie wants to create the maximum profit for her business, she should replace her printing presses at the end of their economic life in 6 years time.**

### Example #2

The youngest intern at the printing company, Paul, is convinced that printing books is a dying industry. Instead of investing in new printing presses, he believes the company should invest in a set of large-scale 3-D printers and shift its business model accordingly. This would be a much larger up-front investment than upgrading to new printing presses, because it would also involve replacing much of the other book-making equipment in the facility. He presents the projected costs and profits below to Sophie for her consideration. Sophie doesn't want to spend

too much time reviewing this proposal, so she only wants to check whether switching to 3-D printers now would be more profitable than keeping the old presses for the next 10 years (note that this is a different analysis period than was used in the previous question). Since Paul could not provide a reasonable estimate for 3-D printer salvage costs, she decides to ignore the salvage value of both alternatives. Using Sophie's methodology, the estimates below, and the data from the previous question, evaluate Paul's proposal.

Paul's 3-D Printer Estimates	
Equipment Purchase:	\$3,250,000
Annual Maintenance:	\$5000/year
Annual Revenue:	\$300,000 in year 1, increasing by 12% per year

Table 6.3 Cost and revenue estimates for replacing the old printing presses with 3-D printers

Step 1: We can find the NPV after 10 years for the old presses by drawing from Table 6.1 we created in the previous question:

$$NPV_{\text{Old}} = \text{Maintenance NPV} + \text{Old Revenue NPV} = -\$18,806 + \$973,307 = \$954,501$$

Step 2: We can find the NPV of the 3-D printers by adding the purchase price to the NPV of the annual maintenance costs (a linear series) and annual revenue (a geometric gradient series):

$$NPV_{3D} = P + A \left[ \frac{(1+i)^N - 1}{i(1+i)^N} \right] + \frac{A}{i-g} \left[ 1 - \left( \frac{1+g}{1+i} \right)^N \right]$$

$$NPV_{3D} = \$827,695$$

**Since the NPV for the old presses (\$954,501) is greater than for the 3-D printers (\$827,695), Sophie would conclude that she should reject Paul's suggestion and keep the old printing presses.** However, since these results are very similar, and annual profits for 3-D printing appear to be rising more quickly than for book printing, Sophie should consider reviewing this analysis with better estimates and a more rigorous procedure.



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## 6.2 Life Cycle Costs

In the Evaluation Methods chapter, we discussed many methods which an investor might use to choose between mutually exclusive projects. The drawback of many of these methods is that they do not account for every economic factor which might impact a project. For example, the Capital Recovery Cost (CRC) method considered the initial price and final salvage value of a project, but did not consider other cash flows such as maintenance costs or revenues. The purpose of **life cycle costing** is to look at *all* of the costs related to a project, so that its cost-effectiveness can be evaluated with greater confidence.

**Life cycle cost (LCC)** is the total cost of a project over its life including planning, design, acquisition, support costs, and decommissioning, as well as any other costs directly attributable to owning or using the asset. These costs encompass the entire life cycle of a project and thus are sometimes referred to as “cradle to grave” costs. They are typically measured or estimated in advance, however, life cycle costing can be applied at any phase of a project’s life. It can provide insight into how the manufacturing, installation, operation, maintenance, or disposal of an existing project can be executed economically. The major advantages of performing life cycle costing at the outset are that cost-effective decisions will have a greater influence on the total cost of a project if they are enacted early, and identifying project costs in advance allows an investor to account for their preferences (performance, reliability, maintenance, etc.) before beginning a project.

When estimating the LCC of a project, we break down the total cost of the project into smaller elements whose costs can be estimated more easily. These costs can be broken down by the significant components of a project, the time when they occur in the project life cycle, and the category of resource cost they fall under (e.g. labour, materials, energy, etc.).

Consider the construction of a new highway. The costs for this project could be broken down into several major components, such as the design process, surveying the route, constructing the embankment, completing the road surface, and long-term maintenance costs (e.g. crack filling and snow removal). Each of these phases could be broken down into smaller cost components. For example, the construction of the road embankment would entail labour costs for labourers and geotechnical staff, material costs for the soil used as well as transportation costs to bring it to the work site, and equipment and fuel costs for operating the necessary construction vehicles. Each of these costs would be attached to a certain time period in the project life cycle. The provincial or federal government constructing the highway must look at the long term cost of maintaining the highway, which will

influence the design process, construction process, and material choice. For example, a certain type of pavement might be more expensive, but would require lower maintenance costs over the road's life cycle.

Once the costs of a project have been broken down to a suitable level of detail, they can be estimated using known cost data. In the early stages of a project, it can be difficult to gather detailed data on the expected costs associated with the project. However, with a common and well-known project type like a highway, the investor could estimate the costs with a fair degree of certainty based on the construction costs of many similar roadways. If the project were a new technology with no similar projects to compare it to, it would need to be estimated roughly using unit cost parameters, such as the expected cost per unit of construction or the cost per labour hour. If life cycle costing is continued throughout the project life, future costs can be estimated based on the costs incurred up to that point, allowing the LCC to be more accurate.

When estimating the life cycle costs for a project, there are three analysis methods to consider:

1. engineering cost method,
2. the analogous cost method, and
3. the parametric cost method.

#### *6.2.1 Engineering Cost Method*

The **engineering cost method** is the most detailed of the three options, and is the preferred method whenever we have access to granular data about the operations and costs of a project. To estimate the entire cost of the project using this method, we treat the LCC as the sum of many components whose costs can be found individually. Each of these component costs is estimated using well established engineering and manufacturing standards, which are expected to be reasonably precise.

This method might be used, for example, in the construction of a new elementary school. The shape of the school, the number and size of the classrooms, and the dimensions of the gymnasium or school library are likely to be different from any other existing school. However, the basic construction materials (concrete, steel beams, metal roof decking) and their associated labour costs can be estimated by an experienced engineering firm after the design for the building has been decided.

#### *6.2.2 Analogous Cost Method*

The **analogous cost method** is less detailed than the engineering cost method, and draws more directly on historical data from analogous projects of a similar type, size, or operational type. For a given project, its cost would be estimated using the costs of other, similar projects, adjusting for variables like size, project location, and inflation.

For example, this method could be used in the design of a steel warehouse building. The design of these buildings is very standardized, and using basic data from suppliers and contractors the lead engineer could estimate the cost of building based on its major dimensions. This method could also be used to estimate the cost of running a

new distribution centre for a large shipping company; if they already operate several similar centres, they could estimate the operational cost of a new facility based on its relative size and expected workload.

Note that this method depends heavily on the existence of a similar project with analogous costs. It cannot be applied to novel projects; for example it could not be applied to the CN Tower, the Panama Canal, or the International Space Station.

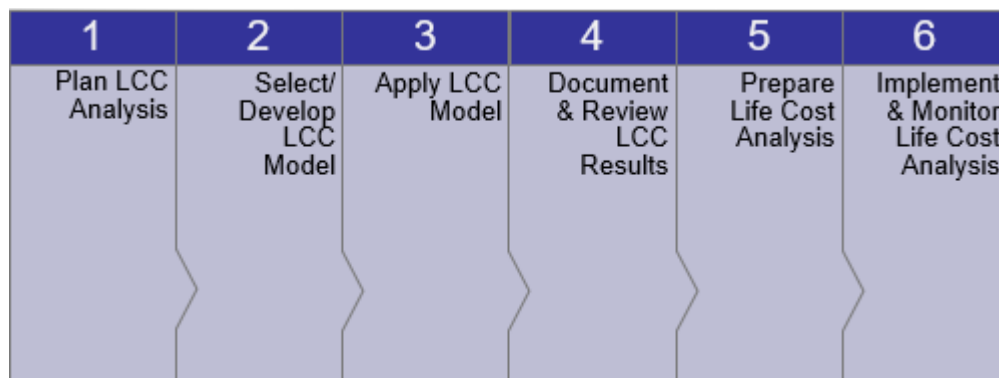
### 6.2.3 Parametric Cost Method

The **parametric cost method** relies on using historical data from certain project components, and scaling that data to the proposed project. This is similar to the analogous method, but This could be accomplished by developing a mathematical regression or progression formula representing the cost estimate.

This method could be used on a project to construct a new provincial highway. The transportation engineers working for the province could use existing data on the cost of roadway construction per kilometre, and then scale that cost by the length of the proposed new highway.

### 6.2.4 Life Cycle Costing Process

Life cycle costing is an ongoing process of analysis throughout the life of a project. When implementing it in a project, we can consider it a 6-step process, which we will discuss in detail here.



#### 1. Plan Analysis

To analyze life cycle costs effectively, the method of analysis must be carefully considered in advance. A well-planned LCC analysis should consider the **purpose** and **scope** of the analysis, as well as its underlying assumptions, limitations, alternatives, required resources, and reporting schedule.

#### 2. Select / Develop LCC Model

A LCC analysis should follow a well-defined model, and not every project is best analyzed by the same methods. For example, the LCC model for evaluating an industrial manufacturing operation with a few large, consistent costs and dependable profits will look much different from the model for a consulting firm which will have many different and fluctuating cash flows. An effective LCC model should break down costs efficiently, identify and

omit redundant cost elements, source relevant data, use a reasonable estimation method, identify the uncertainty of estimates, integrate all costs, and document all assumptions.

### 3. *Apply LCC Model*

The crucial step of this process is applying the chosen LCC model to evaluate the project. Using the assumptions and processes previously selected, the LCC model should obtain data, develop cost estimates, validate results using historic data, identify significant costs, conduct sensitivity analysis, and quantify the differences between multiple alternatives.

### 4. *Document and Review Results*

After the basic processes of LCC analysis are complete, the results must be considered in order to draw meaningful conclusions, and make useful recommendations to the decision makers on the project. All results should be well documented, and should be reviewed critically and thoroughly so that the outcomes, implications, limitations, and uncertainties of the process are clear; the more detailed and well documented this review process is, the more useful it will be to any future or outside party which wishes to learn from your results. It may be fitting to subject the analysis to a formal review, to confirm that its assumptions, scope, model and recommendations of the LCC analysis are acceptable.

### 5. *Prepare Life Cost Analysis (LCA)*

Once the LCC analysis has been completed and reviewed, its results can be used to prepare a **Life Cost Analysis (LCA)** for the project. The LCA is a tool used to control ongoing costs for the project, and is similar to the LCC model, except that it substitutes estimated costs with the actual, ongoing costs of the project. The LCA works in real time to track project costs based on the targets set by the LCC, and may be updated if new data emerges. Using an LCA for a project could be considered analogous to using a flexible budgeting approach as discussed in Chapter 1 since both depend on making cost estimates and then tracking actual project costs based on those estimates.

### 6. *Implement and Monitor LCA*

Since the LCA is an ongoing process, after its creation and implementation it needs to be continuously monitored over the life of the project. The actual performance of a project, as determined by the environmental factors that influence project costs, needs to be closely watched in order to identify potential cost savings and avoid future risk for the project.

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## 6.3 Benefit-Cost Analysis

A **benefit-cost analysis (BCA)** (or cost-benefit analysis) is a decision making tool that attempts to balance the components of a project in order to maximize its net benefits and/or minimize its costs. In this analysis, we try to quantify the costs of project factors that do not truly have monetary costs; for example, loss of life is often assigned a monetary value. These values are then incorporated into the analysis along with other financial costs. Using BCA, decision makers try to either maximize benefits for a set cost, minimize costs for a set level of benefit, or find the most beneficial compromise when both costs and benefits are variable. For any project to be worthwhile, the benefits must exceed the costs.

### 6.3.1 Service Projects vs. Revenue Projects

When evaluating various projects, it is important to draw a distinction between **revenue projects** and **service projects**. A **revenue project** is one that will generate both costs and revenues over its project life. When choosing between multiple revenue projects, you should select the alternative with the highest NPV. Nearly every example we have discussed so far in this text fits the definition of a revenue project.

A **service project** is one that will generate costs, but not revenues, over its project life, or whose revenues are constant regardless of the project alternative chosen. When choosing between multiple service projects, you should select the alternative that performs the service at the lowest cost. For example, a city contracts a company for residential garbage collection. Since the program generates no revenue for the city, when contracting a company to run the collection program, it should choose the business that will run the program at the lowest cost, or do it themselves.

### 6.3.2 Economic Sectors

Importantly, the concept of a service project should not be confused with concepts of services or the service sector. As a significant distinction, all types of economic activity are sometimes divided into three sectors: the primary sector, the manufacturing sector, and the service sector. The **primary sector** involves producing raw materials, and covers activities like mining, fishing, and agriculture. The **manufacturing sector** involves (as you might have guessed) manufacturing products from those raw materials. Finally, the **service sector** supplies services to consumers, where a **service** is anything intangible with economic value.

While the service sector might seem the most abstract of the three economic sectors, it is an enormous part

of the economy for any developed country. Even in Canada, which has significant and well-developed natural resources, the service sector accounts for over 70% of the nation's GDP (Statistics Canada, 2017). The service sector includes a wide-range of industries including retail, transportation, insurance, real estate, health care, and education.

In Chapter 1 we covered how a manufacturing business might set costs for its products. By understanding the direct costs of material and labour for each product, and assigning an appropriate amount of overhead from the other expenses of the business, arriving at an appropriate and competitive price for a product is not overly complicated, at least in theory. However, due to the intangible nature of services, setting prices for them can be a difficult task. For example, imagine an airline that currently charges \$300 for a flight from Saskatoon to Fredericton. If the airline wanted to improve its service by enhancing the variety of in-flight entertainment and refreshment options, or increasing the amount of leg room on each flight, how much more would its customers be willing to pay for these improvements? Conversely, if the airline cut back on these same services in order to reduce its ticket price, would its customers accept the less comfortable conditions for a \$250 ticket? What about for a \$150 ticket?

Since the price of services is so sensitive to the interests of consumers, it can be very responsive to pressures of supply and demand, and determining a service's value can be difficult. This is especially true for services performed in the public sector.

### 6.3.3 *Public Sector vs Private Sector*

You may also hear of the public sector and the private sector. While these share the term “sector”, they are not related to the concept of primary, manufacturing, and service sectors. In this distinction, the **public sector** refers to any business that is owned and operated by the government, while the **private sector** refers to any business that is not owned by the government. Common examples of public sector organizations include (but are not limited to) government ministries, police departments and libraries. Private sector industries are wide ranging often include manufacturing, retail, entertainment, and trades. Sometimes, private sector companies can be hired by governments to perform work in the public sector, or can partner on projects to create what is known as a public private partnership, or P3. Public and private companies may also compete for consumers in the same industries, such as how private shipping companies like UPS compete with national postal services.

It should be noted that there is no consistent rule about which industries or services are in the public sector or private sector, and this will vary depending on a region and its system of government. For example, most communist countries either fully or significantly limit any activity in the private sector, and any country, province, territory, or state may choose to privatize or nationalize different industries depending on their situation (e.g. during WWII, the Canadian government took direct control over most raw material production and manufacturing in order to support the war effort). Even within Canada, services that are public in one province may be private in another; for instance, Saskatchewan is the only province which still owns a public telecommunications company (SaskTel) which competes for consumers with other private companies.

Public sector services often take the form of not-for-profit, service projects. This makes them especially hard to evaluate because the value of their services is not solely financial. For example, how do you decide if one project

is a better investment or more beneficial than another. For this reason, we typically use a benefit-cost analysis to evaluate this kind of project:

### 6.3.4 Framework for Benefit-Cost Analysis

#### Evaluating User Benefits

The first step in performing a BCA is to identify the impacts that a project will have on its users or stakeholders. We identify these impacts as **benefits** if they are positive, and **disbenefits** if they are negative. It can be useful to rank these benefits and disbenefits as primary or secondary, or in a larger hierarchy, in order to understand their relative importance. Primary benefits are those that have a direct impact, while secondary impacts might have an indirect impact. For example, the primary benefit of having a mountain rescue service is that lost skiers are rescued. A secondary benefit is that knowing such a service exists makes people feel safer about going skiing, and thus encourages tourism. When the benefits and disbenefits of a project are directly quantifiable, we find the total value of the benefits for a project with the following equation:

$$\text{Total Benefits (B)} = \sum(\text{Benefits}) - \sum(\text{Disbenefits})$$

To ensure a complete evaluation of benefits, it is necessary to be diligent when identifying all users or stakeholders of a project, and the full range of effects they will experience. For example, consider a city which plans to construct a new athletics stadium. The benefits would include improved facilities for local sports teams (and their fans), a new space for civic, educational, and corporate events, the opportunity to boost tourism and local industry by hosting larger events, new jobs created in constructing and operating the stadium, and potentially many other positive effects. Disbenefits could include traffic delays surrounding the new stadium, or higher ticket prices for events than at the existing facility. In order to properly assess any large project like this, the process of identifying and ranking user benefits must be thorough and deliberate (and can be quite time consuming).

#### Evaluating Costs

After evaluating the benefits of a project, we must also consider the **costs**, which make up the other half of our benefit-cost analysis. Since these costs are often strictly financial, they can be more tangible to calculate than the benefits and disbenefits. It is important to realize that the total costs for a project also incorporate any revenue generated by the project. We find the value of the costs for a project using the following equation:

$$\text{Total Costs (C)} = (\text{Capital Cost}) + (\text{Operating \& Maintenance Cost})$$

With our example of a new athletics stadium, the costs would be fairly straightforward to determine. There would be capital costs for the land acquisition and construction of the new building, operating and maintenance costs for its upkeep, and revenue produced from event ticket sales or rental fees of the facility

#### Quantifying Benefits and Disbenefits

While the equations listed above may make this process look uncomplicated, they gloss over the most difficult aspect of benefit-cost analysis: quantifying the benefits and disbenefits. In section 6.4.1, we gave the example

of a new stadium, which would (among other impacts) create new employment opportunities and cause traffic delays. These benefits and disbenefits are easy to list off, but difficult to assign dollar amounts to. How much is the creation of a new job in the maintenance and operation of the stadium worth to the city? What is the equivalent cost of an added 20-minute delay per commuter during a peak traffic time for the stadium? How will the increased tourism from the stadium translate into increased business for local hotels, restaurants, etc.? These questions do not have straightforward answers. However, since BCA requires us to quantify these kinds of abstract impacts, we need to determine an approach that suits our scenario.

The two basic approaches to quantifying benefits/disbenefits are to determine the **willingness to pay** of consumers, or to set performance metrics for certain impacts of the project.

### **Willingness to pay**

**Willingness to pay** is the highest price that the average consumer would be willing to pay to obtain a product or service, or to avoid a negative outcome. For instance, imagine an open-air music festival where one vendor is selling umbrellas. On one day, the vendor might find they can only charge \$15 per umbrella, and that their sales decline sharply if they raise their prices any higher. However, if a sudden rainstorm strikes the festival, they might be able to raise their prices to \$35 per umbrella, because the willingness to pay of their consumers has been affected by their circumstances.

In this case, we would say that the value of the umbrellas was found using the **revealed preference technique**. This means that the customers of the umbrella vendor were revealing the value of the umbrellas through their purchasing habits. If the price were too high, and the average festival-goer would prefer to save their money and get rained on, then the umbrella would not be worth that amount. If most consumers would gladly exchange the listed price for an umbrella, then the price is acceptable or could be increased. We can also imagine the revealed preference technique working in a reverse situation; if new toll stations were installed on a major commuter bridge, how far out of their way would commuters be willing to travel to avoid the toll? How low would the toll need to be to discourage commuters from taking the detour?

The alternative to the revealed preference technique is the **stated preference technique**. This approach relies on surveying consumers directly about their willingness to pay. The advantage of this technique is that it gives straightforward answers that can be directly applicable to the situation at hand. Would you change laundry detergent brands if there was an alternative \$2 cheaper? How much more would you be willing to pay for internet if your provider upgraded to a high-speed, fiber-optic network? Would a new tax on cigarettes impact your decision to quit (or begin) smoking? Would increased ticket prices for speeding cause you to change your driving habits?

The drawback of this approach is that consumers stated preferences do not always line up well with their actual preferences. For the examples above, a consumer might say they'd switch to a cheaper detergent, but in practice stay with their usual brand out of habit. They might also underestimate their preference for faster internet, overestimate their ability to quit smoking, and quickly forget about the increased speeding fines (until they get written their first ticket). In short, the stated preference technique can be useful for determining willingness to pay, but is not a foolproof option.



## Performance Metrics

Another commonly used approach for determining the value of a cost or benefit is to set performance metrics for the impacts of a project. For example, when transportation engineers compare alternative roadway designs, they calculate the expected number of crashes for each design, and the severity of those crashes. Most often, the crashes are classified as either “fatal” crashes, “injury” crashes, or “property damage only (PDO)” crashes, and a certain dollar value is assigned to each type of crash based the severity of the damage done (e.g. a “PDO” crash might be valued around \$1000, and a fatal crash might be valued around \$1,000,000). Then, by multiplying these values by the expected number of crashes, the relative cost savings (also representing human safety) can be compared to the price to inform whether a project is worthwhile.

For a project with environmental impacts, there are many possible routes for evaluation. If a project generates some kind of pollutant, the metric could be based on the cost of removing that pollutant from the ecosystem, or on the contributing cost to the healthcare system for any negative health outcomes it encourages (e.g. more air pollution might lead to increased respiratory problems). Carbon pricing methods like a carbon tax or a cap-and-trade system are examples of a government attaching a cost metric to an environmental impact (in this case, CO<sub>2</sub> emissions) in order to have businesses consider their environmental effects as a tangible cost.

Assigning performance metrics to determine the value of a cost or benefit is not an exact science, and for any given benefit there are likely to be many different ways of quantifying it. The important factor is to quantify the effect on the basis of a logically related cost, and to arrive at a performance metric that scales appropriately with the importance of the impact.

## Determining Social Discount Rate

In Chapter 5 we found that many financial evaluation methods are very sensitive to the discount rate applied during analysis. Setting an appropriate interest rate or MARR is a vital step in determining the value of any project. Benefit cost analysis is no different. When performing analysis for a services project, we refer to the discount rate as a **social discount rate**.

The social discount rate is set slightly differently than other discount rates we have discussed previously. There are two basic approaches to setting the rate, depending on whether the project has private sector involvement:

1. For a fully public sector project, **the social discount rate should reflect only the prevailing government borrowing rate.**
2. For a project with private counterparts, **the social discount rate should reflect the rate that could have been earned had the funds not been removed from the private sector.**

Once the social discount rate has been set, it is used in the same way as the discount rate for any other project: to discount cash flows from different points in the project to a common time frame.

### 6.3.5 Evaluating Results

Once the benefits and costs of a project have been determined, there are two commonly used indicators for evaluating the project: the **benefit cost ratio** and the **profitability index**.

### Benefit Cost Ratio

The **benefit cost ratio** for a project is the present value of the project's benefits divided by the present value of its costs- ensuring all costs and benefits are compared during the same time scale is an important consideration of this process. Any future amounts can be converted to present values using the determined social discount rate.

$$\text{Benefit Cost Ratio} = \frac{\text{PV of Benefits}}{\text{PV of Costs}}$$

If the benefit cost ratio is greater than 1 (i.e. the benefits are greater than the costs), then the project should be accepted. If it is less than 1, then the project should be rejected.

### Profitability Index

The **profitability index** (PI) for a project is slightly different from the benefit cost ratio, since it considers costs and benefits on an annual basis and incorporates the capital investment cost of project separately. Again, the costs and benefits associated with the project must all be considered at their present values.

As with the benefit cost ratio, if the PI is greater than 1, the project should be accepted. Otherwise the project should be rejected.

### Comparing Multiple Projects

Suppose you were performing benefit-cost analyses to find the best alternative for the expansion of an existing airport. You considered five alternatives, which returned benefit cost ratios of 1.2, 0.8, 1.15, 1.3, and 0.95. You might assume that the alternative with the 1.3 benefit-cost ratio is the best alternative, since it has the highest ratio. However, the best approach in this situation is to use an incremental analysis technique, consisting of the following steps:

1. Eliminate any alternatives with benefit-cost ratios less than 1.
2. Arrange the remaining alternatives from lowest cost to highest cost.
3. Compute the incremental differences from the paired alternatives.
4. Compute the benefit cost ratio again based on the incremental benefits and costs.
5. Compare the chosen alternative to the next alternative.

This technique is best demonstrated using an example. Let's look at the airport expansion example from above, but in greater detail. Using the data provided below, let's walk through the steps of this analysis.

Alternative	Initial Investment	Benefits	Costs	Benefit/ Cost Ratio
A	\$1,000,000	\$48,000	\$40,000	1.2
B	\$1,500,000	\$48,000	\$60,000	0.8
C	\$800,000	\$40,250	\$35,000	1.15
D	\$1,250,000	\$58,500	\$45,000	1.3
E	\$750,000	\$33,250	\$35,000	0.95

1. *Eliminate any alternatives with benefit-cost ratios less than 1.*

This leaves us with alternatives A, C, and D.

2. *Arrange the remaining alternatives from lowest cost to highest cost.*

This leaves us with the following table of results:

Alternative	Initial Investment	Benefits	Costs	Benefit/ Cost Ratio
C	\$800,000	\$40,250	\$35,000	1.15
A	\$1,000,000	\$48,000	\$40,000	1.2
D	\$1,250,000	\$58,500	\$45,000	1.3

3. *Compute the incremental differences from the paired alternatives.*

4. Compute the benefit cost ratio again based on the incremental benefits and costs.

5. Compare the chosen alternative to the next alternative.